The purpose of this ER is to prove Prop. 3.27, which we proved in class using DA, by using mod arithmetic.

▷. **Proposition 3.27**. If n is an integer, then 3 divides $n^3 - n$.

►. Finish the below *Almost finished Proof* by doing case 3. Be sure to follow the Writing Guidelines. In your case 3, the r that you choose is the integer that goes into the *Almost Finished Proof* in (2) where the ?? currently are. ⟨So just write out case 3, as if you were inserting it into the proof.⟩

Almost Finished Proof. Let $n \in \mathbb{Z}$. By definition of modulo congruence, 3 divides $n^3 - n$ is equivalent to

$$n^3 \equiv n \pmod 3. \tag{1}$$

Also,

$$n \equiv r \pmod 3$$

for a unique integer r such that

$$r \in \{0, 1, \underline{\quad ?? \quad}\}. \tag{2}$$

We shall show Prop. 3.27 by showing (1) holds for each of the 3 possible case of r in (2).

For case 1, let $r = 0$. In this case

$$n \equiv 0 \pmod 3, \tag{3}$$

which implies by mod arithmetic ⟨multiply⟩ that $n^3 \equiv 0^3 \pmod 3$ and so

$$n^3 \equiv 0 \pmod 3 \tag{4}$$

and by the transitivity of mod congruence, (4), and (3) we get

$$n^3 \equiv n \pmod 3.$$

Thus (1) holds in this case. This complete case 1.

For case 2, let $r = 1$. In this case

$$n \equiv 1 \pmod 3, \tag{5}$$

which implies by mod arithmetic ⟨mulitply⟩ that $n^3 \equiv 1^3 \pmod 3$ and so

$$n^3 \equiv 1 \pmod 3 \tag{6}$$

and by the transitivity of mod congruence, (6), and (5) we get

$$n^3 \equiv n \pmod 3.$$

Thus (1) holds in this case. This complete case 2.

This case for you to do. Hint. Good start is: For case 3, let $r =$

We have just shown that (1) holds for each possible case. Thus this completes the proof. □