Evaluation of Proof Exercise

Following the instructions for (linked) *Evaluation of Proofs* exercises (which also are posted on the course homework page), evaluate the below justification of the given conjecture.

►. **Conjecture B**. For all real numbers x and y, if x is irrational and y is rational, then $x + y$ is irrational.

Proposed Proof. We will proof Conjecture B is true by using proof by contradiction. By way of contradiction, assume that x and y are real numbers such that where

$$x \notin \mathbb{Q}, \tag{1}$$

$$y \in \mathbb{Q} \tag{2}$$

and

$$x + y \in \mathbb{Q}. \tag{3}$$

Since both $x + y$ and y are rational (cf. (2), (3)) and the rational numbers are closed under subtraction, we have

$$(x + y) - y \in \mathbb{Q}. \tag{4}$$

However, $(x + y) - y = x$, and hence we can conclude from (4) that

$$x \in \mathbb{Q}. \tag{5}$$

Note (5) is a contradiction to the assumption in (1) that $x \notin \mathbb{Q}$. Hence assuming that Conjecture B is false leads to a contradition.

We have proven Conjecture B must be true, i.e., for all real numbers x and y, if x is irrational and y is rational, then $x + y$ is irrational. □

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