The symbol for the rational numbers is $\mathbb{Q}$ while the symbol for the irrational numbers is $\mathbb{R} \setminus \mathbb{Q}$.
So you can express that x is an irrational number by $x \notin \mathbb{Q}$ or by $x \in \mathbb{R} \setminus \mathbb{Q}$.
Recall for any sets R and Q, the set R set minus Q is the set $R \setminus Q \overset{\text{def.}}{=} \{x \in R : x \notin Q\}$.
Note the difference in direction in the slash for set minus ($R \setminus Q$) and quotient of numbers ($1/2 = 0.5$).

►. **Proposition 3.19**. [TS textbook §3.3 page 123]. For all real numbers x and y, if x is rational and $x \neq 0$ and y is irrational, then xy is irrational.

▷. Symbolically written, Proposition 3.19 says

$$\big(\forall\,(x,y) \in \mathbb{R}^2\big)\ [\ (\ x \in \mathbb{Q} \wedge x \neq 0 \wedge y \notin \mathbb{Q}\ ) \implies xy \notin \mathbb{Q}\ ] \tag{1}$$

1. Complete the next sentence (write out the whole sentence, filling in the blank).

   Proposition 3.19 gives that the irrational numbers are closed under the operation of multiplication by ____.

rmk. We have discussed closure properties of number systems (e.g., $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{R} \setminus \mathbb{Q}$) under cetain operations (e.g., addition, multiplication, division by a nonzero number). Finally we see the irrational real numbers are closed under some operation.

2. Using that $\sim [P \implies Q] \equiv [P \wedge (\sim Q)]$ ⟨idea: negation of a promise is a lie⟩, symbolically write a negation (denial) if the statement in (1). Your answer can not contain the negation symbol $\sim$ ⟨nor any variant of the $\sim$ symbol⟩. Your answer can contain the symbols $\in$ and $\notin$.

hint. You may (and probably will) use Proposition 3.19 in the ER's from §3.3 and beyond.

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