Recall (a.k.a., hint) you may use either below (equivalent) formulation as the definition of rational number.
Def. 1. A real number x is <u>rational</u> provided

$$(\exists a \in \mathbb{Z})\,(\exists b \in \mathbb{Z})\left[\; x = \frac{a}{b} \;\wedge\; b \neq 0 \;\right].$$

Def. 2. A real number x is <u>rational</u> provided

$$(\exists a \in \mathbb{Z})\,(\exists b \in \mathbb{N})\left[\; x = \frac{a}{b} \;\right]$$

Def. 1 appeared on p11. Def. 2 is often easier to work with.

►. **Theorem 1**. If $p, q \in \mathbb{Q}$ with $p < q$, then there exists $x \in \mathbb{Q}$ with $p < x < q$.

1. Symbolically write Theorem 1. Hint. Your answer should be of the form

$$\big(\,\forall\,(p,q) \in \mathbb{Q}^2\,\big)\;\;[p < q \quad\Longrightarrow\quad (\exists x \in \mathbb{Q})\,[\text{?what goes here?}]\;].$$

2. Prove Theorem 1. You may use the closure properites of $\mathbb{Q}$ that we covered in Chapter 1.

Hint: let geometry lead your thinking land but make sure you justify in your proof by using algebra.

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