Evaluation of Proof Exercise

Following the instructions for (linked) *Evaluation of Proofs* exercises (which also are posted on the course homework page), evaluate the below justification of the given conjecture.

►. **Conjecture 1**. For all integers a, b, and c with $a \neq 0$, if a divides bc then a divides b or a divides c.

Hint. Symbolically written: $\big(\forall\, (a, b, c) \in \mathbb{Z}^{\neq 0} \times \mathbb{Z} \times \mathbb{Z}\big)\ [\ (\ a \mid (bc)\ ) \implies (\ a \mid b\ \vee a \mid c\ )\ ]$

Proposed Proof. We assume that a, b, and c are integers such that $a \neq 0$ and that a divides bc. So, there exists an integer k such that

$$bc \ = \ ka. \tag{1}$$

We now factor k as $k = mn$, where m and n are integers. We then see, from (1) that

$$bc = mna. \tag{2}$$

Equation (2) implies that

$$b \ = \ ma \qquad \text{or} \qquad c \ = \ na. \tag{3}$$

and hence, by (3) and definition of divides, $a \mid b$ or $a \mid c$. □

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