Pin: Variant of **2.3.7**.
Name: Sundstrom §2.3 p63. Math 300

Closure Explorations | Explorations and Activities Exercise

In this exercise, you may use (without proof) the following prior results from the (linked) Ch. 1 class handout.

Previously Shown Results

Lemma SEE. If x is an even integer and y is an even integer, then $x + y$ is an even integer.
Lemma SEO. If x is an even integer and y is an odd integer, then $x + y$ is an odd integer.
Lemma SOO. If x is an odd integer and y is an odd integer, then $x + y$ is an even integer.
Lemma PEA. If x is an even integer and y is an integer, then $x \cdot y$ is an even integer.
Lemma POO. If x is an odd integer and y is an odd integer, then $x \cdot y$ is an odd integer.

Division Algorithm

Division Algorithm. For all $n \in \mathbb{N}$ and $a \in \mathbb{Z}$, there exist unique integers q and r so that

$$a = nq + r \qquad \text{and} \qquad 0 \leq r < n \ .$$

In §1.1, we studied closure properties of standard number systems (e.g. $\mathbb{Z}$ and $\mathbb{Q}$, see p. 11–12 and 32).
We can extend the closure idea to other subsets S of real numbers. We say that

- A subset S of real numbers is closed under addition provided that if x and y are in the set S, then $x + y \in S$.
- A subset S of real numbers is closed under multiplication provided that if x and y are in the set S, then $x \cdot y \in S$.
- A subset S of real numbers is closed under subtraction provided that if x and y are in the set S, then $x - y \in S$.

▶. Consider the below subsets of of the real numbers defined by: ⟨note T is not in set builder notation⟩

O is the set of all odd integers

E is the set of all even integers

$$T = \{3n + 2 \in \mathbb{Z} \colon n \in \mathbb{Z}\} \overset{\text{i.e.}}{=} \{\ldots, -7. - 4, -1, 2, 5, 8, \ldots\}.$$

Below the dotted line, answer each question YES or NO. Then justify your Yes/No answer by either

- (for yes) explaining which and why prior class result ⟨from above box⟩ say yes (no formal proof needed)
- (for no) providing an counterexample showing the answer is no.

O.1. Is O closed under addition?

O.2. Is O closed under multiplication?

O.3. Is O closed under substraction?

E.1. Is E closed under addition?

E.2. Is E closed under multiplication?

E.3. Is E closed under substraction?

T.1. Is T closed under addition?

T.2. Is T closed under multiplication?

T.3. Is T closed under substraction?

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