Important Exercise | Explorations and Activities Exercise

►. Let's compare the below Theorem A and Theorem B.

Theorem A. Let x and y be integers.

$$\text{If } x \cdot y \text{ is even, then } x \text{ is even or } y \text{ is even.} \tag{A}$$

Theorem B. Let x and y be integers.

$$\text{If } x \text{ is odd and } y \text{ is odd, then } x \cdot y \text{ is odd.} \tag{B}$$

▹. Notice that statement in (A) is of the basic ⟨unquantified⟩ symbolic form of

$$P \implies (Q \vee R) \tag{0}$$

where P is "$x \cdot y$ is even", Q is "x is even", and R is "y is even" ⟨with the understanding $x, y \in \mathbb{Z}$⟩.

1. Write the contrapositive of the conditional statement in (0) in basic ⟨unquantified⟩ symbolic form. ⟨Just write the contrapositive of the statement in (0). Do not use DeMorgan (yet). Your solution should have the atoms (i.e.: P, Q, R) joined by logical operators/connectives (so look similar to (0)). Your solution should not have: x, y, English words.⟩
2. Using one of De Morgan's Laws, write a logically equivalent statement to your statement in part 1 in basic ⟨unquantified⟩ symbolic form. ⟨Your solution should have the atoms (i.e.: P, Q, R) joined by logical operators/connectives (so should look similar to (0)). Your solution should not have: x, y, English words.⟩
3. Using the result from the previous parts of this problem,
explain why the statement in (A) is logically equivalent to the statement in (B).

. . . .

Important lesson learned from this problem.
We just showed (A) and (B) are logically equivalent statements. So to prove (A), we can either prove (A) (as it is written) or prove the logically equivalent (B) (which is the previously shown (in §1.2) result Lemma POO).

Proof of Thm. A. We will proof Theorem A by contrapositive. Applying DeMorgan, the contrapositve of Thm. A is: if x and y are odd integers then xy is an odd integer. Let x and y be odd integers. Thus we need to show that that xy is an odd integer.
Since x and y are odd integers, by Lemma POO, xy is an odd integer.
We have just shown that contrapositive of Thm. A is true. Thus Theorem A is true. □