

▷. **Definition.** Let A point $(x_0, y_0) \in \mathbb{R}^2$ is:

- inside the circle whose equation is $(x - h)^2 + (y - k)^2 = r^2$ provided $(x_0 - h)^2 + (y_0 - k)^2 < r^2$
- on the circle whose equation is $(x - h)^2 + (y - k)^2 = r^2$ provided $(x_0 - h)^2 + (y_0 - k)^2 = r^2$
- outside the circle $(x - h)^2 + (y - k)^2 = r^2$ provided $(x_0 - h)^2 + (y_0 - k)^2 > r^2$.

▶. **Theorem 1.** Each point on or inside the circle whose equation is

$$(x - 1)^2 + (y - 2)^2 = 4$$

is also inside the circle whose equation is

$$x^2 + y^2 = 26 .$$

Symbolically written: $(\forall (x, y) \in \mathbb{R}^2) [(x - 1)^2 + (y - 2)^2 \leq 4 \implies x^2 + y^2 < 26]$.

.....

1. Let x, h, r be real numbers and $r > 0$. For each of the following statements, indicate whether the statement is TRUE or FALSE. No justification needed.

- (a) If $(x - h)^2 \leq r^2$, then $0 \leq |x - h| \leq r$.
- (b) If $(x - h)^2 \leq r^2$, then $-r \leq x - h \leq r$.
- (c) If $-3 \leq x \leq 2$, then $0 \leq x^2 \leq 4$.
- (d) If $-3 \leq x \leq 2$, then $0 \leq x^2 \leq 9$.

2. Prove Theorem 1 algebraically by using (in)equalities, similar to the class example involving the distance between points in \mathbb{R}^2 . Do **not** use calculus. Do not argue geometrically but rather use geometric idea to form your Thinking Land.

▷. **Hint.** If $(x - 1)^2 + (y - 2)^2 \leq 4$, then $0 \leq (x - 1)^2 \leq 4$ since

$$0 \leq (x - 1)^2 = (x - 1)^2 + 0 \leq (x - 1)^2 + (y - 2)^2 \leq 4.$$

.....