Recall the definition of divides. (See Ch. 1 handout.)

Def. A nonzero integer n **divides** an integer b, denoted $n|b$, provided there exists $k \in \mathbb{Z}$ such that $b = nk$. §3.1 p82

So $n \in \mathbb{Z} \setminus \{0\}$ divides $b \in \mathbb{Z}$ provided $(\exists k \in \mathbb{Z})\ [b = nk]$.

Remarks.

- Note 5 divides 10, also written $5|10$, since $10 = 5 \cdot 2$ and $2 \in \mathbb{Z}$.
- The expression "7 divides $(b+1)$" can be written as $7|(b+1)$.
- Note $7|b+1$ is wrong (and makes absolutely no sense) since $7|b$ is a statement while 1 is a real number (you cannot add a statement and a number). Lesson learned: don't forget your paretbeses,

Compare the definition of divides to the definitions of even and odd integer.

Def. An integer b is an **even integer** provided that there exists an $k \in \mathbb{Z}$ such that $b = 2k$. §1.2 p15

So $b \in \mathbb{Z}$ is even provided $(\exists k \in \mathbb{Z})\ [b = 2k]$.

Def. An integer b is an **odd integer** provided there exists an $k \in \mathbb{Z}$ such that $b = 2k + 1$. §1.2 p15

So $b \in \mathbb{Z}$ is odd provided $(\exists k \in \mathbb{Z})\ [b = 2k + 1]$.

▷. **Theorem 1**. If x and y are integers such that 5 divides x and 5 divides y, then 5 divides $2x + 3y$.

Symbolically written: $(\forall x \in \mathbb{Z})\ (\forall y \in \mathbb{Z})\ [\ (5|x\ \wedge\ 5|y) \implies 5|\,(2x + 3y)\ ]$.

►. Prove Theorem 1.

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