Evaluation of Proof Exercise

Following the instructions for (linked) *Evaluation of Proofs* exercises (which also are posted on the course homework page), evaluate the below justification of the given conjecture.

Recall the definition of divides. (See Ch. 1 handout.)

Def. A nonzero integer n **divides** an integer b, denoted $n|b$, provided there exists $k \in \mathbb{Z}$ such that $b = nk$. §3.1 p82

So $n \in \mathbb{Z} \setminus \{0\}$ divides $b \in \mathbb{Z}$ provided $(\exists k \in \mathbb{Z})\ [b = nk]$.

Remarks.

- Note 5 divides 10, also written $5|10$, since $10 = 5 \cdot 2$ and $2 \in \mathbb{Z}$.
- The expression "7 divides $(b+1)$" can be written as $7|(b+1)$.
- Note $7|b+1$ is wrong (and makes absolutely no sense) since $7|b$ is a statement while 1 is a real number (you cannot add a statement and a number). Lesson learned: don't forget your paretbeses,

Compare the definition of divides to the definitions of even and odd integer.

Def. An integer b is an **even integer** provided that there exists an $k \in \mathbb{Z}$ such that $b = 2k$. §1.2 p15

So $b \in \mathbb{Z}$ is even provided $(\exists k \in \mathbb{Z})\ [b = 2k]$.

Def. An integer b is an **odd integer** provided there exists an $k \in \mathbb{Z}$ such that $b = 2k + 1$. §1.2 p15

So $b \in \mathbb{Z}$ is odd provided $(\exists k \in \mathbb{Z})\ [b = 2k + 1]$.

►. **Conjecture 1**. If x and y are integers such that 5 divides x and 5 divides y, then 5 divides $x + y$.

Symbolically written: $(\forall x \in \mathbb{Z})\ (\forall y \in \mathbb{Z})\ [\ (5|x\ \wedge\ 5|y) \implies 5|\,(x+y)\ ]$.

Proposed Proof. Let x and y be integers such that $5|x$ and $5|y$. We shall show that $5|\,(x+y)$.

Since $5|x$ and $5|y$, by definiton of divides, there exists $k_x, k_y \in \mathbb{Z}$ such that

$$x = 5k_x \qquad (1)$$

$$y = 5k_y. \qquad (2)$$

By (1) and (2), followed by algebra, we have

$$\begin{aligned} x + y &= 5k_x + 5k_y \\ &= 5\,(k_x + k_y) \end{aligned}$$

and letting $j = k_x + k_y$

$$= 5j.$$

Note $j \in \mathbb{Z}$ since $k_x, k_y \in \mathbb{Z}$ and the integers are closed under addition.

We have just shown that $x + y = 5j$ for some integer j. So by definition of divides, $5|\,(x+y)$. □

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