The purpose of this exercise is to prove that the rational numbers $\mathbb{Q}$ are closed under addition.

▹. **Theorem 1**. If $x, y \in \mathbb{Q}$, then $x + y$ is a rational number.

hint. Symbolically written: $(\forall x \in \mathbb{Q})\ (\forall y \in \mathbb{Q})\ [\ x + y \in \mathbb{Q}\ ]$

▹. In class we gave 2 (equivalent) definition of rational numbers $\mathbb{Q}$. One defintion is

$$\mathbb{Q} = \left\{ x \in \mathbb{R}\ :\ x = \frac{a}{b} \text{ for some } a, b \in \mathbb{Z} \text{ with } b \neq 0 \right\}. \qquad \text{(Do Not Use)}$$

The other (simplier) definition is

$$\mathbb{Q} \ = \ \left\{ x \in \mathbb{R}\ :\ x = \frac{a}{b} \text{ for some } a \in \mathbb{Z} \text{ and } b \in \mathbb{N} \right\}. \qquad \text{(USE)}$$

►. Prove Theorem 1 by using the definition of rational numbers in (USE).

You may not use the closure properties of $\mathbb{Q}$ ⟨since we are proving a closure propery of $\mathbb{Q}$⟩.

However, you may use the closure propetries of $\mathbb{N}$ and $\mathbb{Z}$.

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