Def. A conjecture is a statement that we believe is plausible (but we do not have a proof for it ... yet).

Def. A counterexample to a conjecture is an example that shows the conjecture is false.

Evaluation of Proof Exercise

Following the instructions for (linked) *Evaluation of Proofs* exercises (which also are posted on the course homework page), evaluate the below justification of the given conjecture.

►. **Conjecture 1.** If x and y are odd integers, Then $xy + 7$ is an even integer.

Note symbolically: $(\forall x \in \mathbb{Z})\ (\forall y \in \mathbb{Z})\ [\ (x \text{ is odd } \wedge\ y \text{ is odd }) \implies xy + 7 \text{ is even }]$

Proposed Proof. Let x and y be odd integers. We will show that $xy + 7$ is an even integer.

Since x is an odd integer, by definition of odd integer, there exists $k_x \in \mathbb{Z}$ such that

$$x = 2k_x + 1. \tag{1}$$

Since y is an odd integer, by definition of odd integer, there exists $k_y \in \mathbb{Z}$ such that

$$y = 2k_y + 1. \tag{2}$$

Note 7 is an odd integer, by definition of odd integer, since $7 = 2(3) + 1$. Using (1) and (2) and then using algebra we get

$$\begin{aligned} xy + 7 &= (2k_x + 1)\ (2k_y + 1) + 7 \\ &= 4k_xk_y + 2k_x + 2k_y + 1 + 7 \\ &= 4k_xk_y + 2k_x + 2k_y + 8 \\ &= 2\,(2k_xk_y + k_x + k_y + 4) \end{aligned}$$

and letting $j = 2k_xk_y + k_x + k_y + 4$

$$= 2j.$$

Note $j \in \mathbb{Z}$ since $2, 4, k_x, k_y \in \mathbb{Z}$ and the integers are closed under multiplication and addition.

We have just show $xy + 7 = 2j$ for some $j \in \mathbb{Z}$. Thus $xy + 7$ is even by definition of even. □

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