'Ch 6. Functions

6.1 Introduction to Functions

p281 - 294

Definition. A **function** from a set A to a set B is a rule that associates with each element x of the set A exactly one element of the set B. A function from A to B is also called a **mapping** from A to B.

Definition. Let $f : A \to B$. (This is read, "Let f be a function from A to B.") The set A is called the **domain** of the function f, and we write $A = \text{dom}(f)$. The set B is called the **codomain** of the function f, and we write $B = \text{codom}(f)$.

If $a \in A$, then the element of B that is associated with a is denoted by $f(a)$ and is called the **image of a under f**. If $f(a) = b$, with $b \in B$, then a is called a **preimage of b under f**.

Definition. Let $f : A \to B$. The set $\{f(x) \mid x \in A\}$ is called the **range of the function f** and is denoted by range (f). The range of f is sometimes called the **image of the function f** (or the **image of A under f**).

6.3 Injections, Surjections, and Bijections

p 307 - 323

Definition. Let $f : A \to B$ be a function from the set A to the set B. The function f is called an **injection** provided that

for all $x_1, x_2 \in A$, if $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$.

When f is an injection, we also say that f is a **one-to-one function**, or that f is an **injective function**.

Definition. Let $f : A \to B$ be a function from the set A to the set B. The function f is called a **surjection** provided that the range of f equals the codomain of f. This means that

for every $y \in B$, there exists an $x \in A$ such that $f(x) = y$.

When f is a surjection, we also say that f is an **onto function** or that f maps **A onto B**. We also say that f is a **surjective function**.

Definition. A **bijection** is a function that is both an injection and a surjection. If the function f is a bijection, we also say that f is **one-to-one and onto** and that f is a **bijective function**.

Let $f : A \to B$.

"The function f is an injection" means that

- For all $x_1, x_2 \in A$, if $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$; or
- For all $x_1, x_2 \in A$, if $f(x_1) = f(x_2)$, then $x_1 = x_2$.

"The function f is not an injection" means that

- There exist $x_1, x_2 \in A$ such that $x_1 \neq x_2$ and $f(x_1) = f(x_2)$.

Let $f : A \to B$.

"The function f is a surjection" means that

- ~~range(f) = codom(f)~~ $= B$; or not helpful
- helpful For every $y \in B$, there exists an $x \in A$ such that $f(x) = y$.

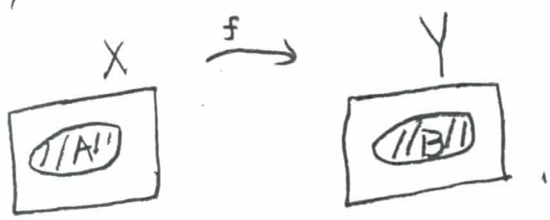
"The function f is not a surjection" means that

- range$(f) \neq$ codom(f); or
- There exists a $y \in B$ such that for all $x \in A$, $f(x) \neq y$.

Def Consider a function $f : X \to Y$.

Let $A \subseteq X$ and $B \subseteq Y$.

Pictorially we have



Then we define the following sets.

$$f[A] \overset{\text{def}}{:=} \{ f(a) \in Y \mid a \in A. \}.$$

$$f^{-1}[B] \overset{\text{def}}{:=} \{ x \in X \mid f(x) \in B \}.$$

note [ ] brackets.

note If $a \in A$ then $f(a) \in Y$.

If $D \subset X$, then $f[D] \subset Y$.

Key	$y \in f[A] \iff \exists\, a \in A$ st. $y = f(a)$.
	$x \in f^{-1}[B] \iff f(x) \in B$

6.4 Composition of Functions

> **Definition.** Let A, B, and C be nonempty sets, and let $f : A \to B$ and $g : B \to C$ be functions. The **composition of f and g** is the function $g \circ f : A \to C$ defined by
>
> $$(g \circ f)(x) = g(f(x))$$
>
> for all $x \in A$. We often refer to the function $g \circ f$ as a **composite function**.

Pictorially $\boxed{g \circ f}$ is $A \xrightarrow{f} B \xrightarrow{g} C$