Ch 5 Set Theory

p55 §2.3

Definition. Two sets, A and B, are **equal** when they have precisely the same elements. In this case, we write $A = B$. When the sets A and B are not equal, we write $A \neq B$.

The set A is a **subset** of a set B provided that each element of A is an element of B. In this case, we write $A \subseteq B$ and also say that A is **contained** in B. When A is not a subset of B, we write $A \not\subseteq B$.

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Definition. Let A and B be two sets contained in some universal set U. The set A is a **proper subset** of B provided that $A \subseteq B$ and $A \neq B$. When A is a proper subset of B, we write $A \subset B$.

Unambigous

- $A = B \iff [(x \in A \Rightarrow x \in B) \wedge (x \in B \Rightarrow x \in A)]$
- $A \subseteq B \iff [x \in A \Rightarrow x \in B] \overset{\text{say}}{\iff}$ A is a subset of B.
- $A \subsetneq B \iff [A \subseteq B \wedge A \neq B] \overset{\text{say}}{\iff}$ A is a proper subset of B,

Ambigous is $A \subset B$

- in text book $A \subset B \iff A \subsetneq B$
- in class often $A \subset B \iff A \subseteq B$.
- Textbook HW uses textbook notation. On Exam, we will use unambigous

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Theorem 5.2. *Let A and B be subsets of some universal set U. Then $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.*

→ To show $A = B$, this is Key method of proof

§5.2 p236

Definition. Let A and B be subsets of the universal set U. The sets A and B are said to be **disjoint** provided that $A \cap B = \emptyset$.

§5.1 p223

Definition. The number of elements in a finite set A is called the **cardinality** of A and is denoted by card (A).

§5.1 p222

Definition. If A is a subset of a universal set U, then the set whose members are all the subsets of A is called the **power set** of A. We denote the power set of A by $\mathcal{P}(A)$. Symbolically, we write

$$\mathcal{P}(A) = \{X \subseteq U \mid X \subseteq A\}.$$

That is, $X \in \mathcal{P}(A)$ if and only if $X \subseteq A$.

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Definition. Let A and B be subsets of some universal set U. The **intersection** of A and B, written $A \cap B$ and read "A intersect B," is the set of all elements that are in both A and B. That is,

$$A \cap B = \{x \in U \mid x \in A \text{ and } x \in B\}.$$

The **union** of A and B, written $A \cup B$ and read "A union B," is the set of all elements that are in A or in B. That is,

$$A \cup B = \{x \in U \mid x \in A \text{ or } x \in B\}!.$$

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Definition. Let A and B be subsets of some universal set U. The **set difference** of A and B, or **relative complement** of B with respect to A, written $A - B$ and read "A minus B" or "the complement of B with respect to A," is the set of all elements in A that are not in B. That is,

$$A - B = \{x \in U \mid x \in A \text{ and } x \notin B\}.$$

The **complement** of the set A, written A^c and read "the complement of A," is the set of all elements of U that are not in A. That is,

$$A^c = \{x \in U \mid x \notin A\}.$$

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Theorem 5.18 (Algebra of Set Operations). *Let A, B, and C be subsets of some universal set U. Then all of the following equalities hold.*

Properties of the Empty Set and the Universal Set	$A \cap \emptyset = \emptyset$ $A \cup \emptyset = A$	$A \cap U = A$ $A \cup U = U$
Idempotent Laws	$A \cap A = A$	$A \cup A = A$
Commutative Laws	$A \cap B = B \cap A$	$A \cup B = B \cup A$
Associative Laws	$(A \cap B) \cap C = A \cap (B \cap C)$ $(A \cup B) \cup C = A \cup (B \cup C)$	
Distributive Laws	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	

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Theorem 5.20. *Let A and B be subsets of some universal set U. Then the following are true:*

Basic Properties	$(A^c)^c = A$ $A - B = A \cap B^c$
Empty Set and Universal Set	$A - \emptyset = A$ *and* $A - U = \emptyset$ $\emptyset^c = U$ *and* $U^c = \emptyset$
De Morgan's Laws	$(A \cap B)^c = A^c \cup B^c$ $(A \cup B)^c = A^c \cap B^c$
Subsets and Complements	$A \subseteq B$ *if and only if* $B^c \subseteq A^c$

§5.4
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Definition. Let A and B be sets. An **ordered pair** (with first element from A and second element from B) is a single pair of objects, denoted by (a, b), with $a \in A$ and $b \in B$ and an implied order. This means that for two ordered pairs to be equal, they must contain exactly the same objects in the same order. That is, if $a, c \in A$ and $b, d \in B$, then

$$(a, b) = (c, d) \text{ if and only if } a = c \text{ and } b = d.$$

The objects in the ordered pair are called the **coordinates** of the ordered pair. In the ordered pair (a, b), a is the **first coordinate** and b is the **second coordinate**.

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Theorem 5.25. *Let A, B, and C be sets. Then*

1. $A \times (B \cap C) = (A \times B) \cap (A \times C)$
2. $A \times (B \cup C) = (A \times B) \cup (A \times C)$
3. $(A \cap B) \times C = (A \times C) \cap (B \times C)$
4. $(A \cup B) \times C = (A \times C) \cup (B \times C)$
5. $A \times (B - C) = (A \times B) - (A \times C)$
6. $(A - B) \times C = (A \times C) - (B \times C)$
7. *If $T \subseteq A$, then $T \times B \subseteq A \times B$.*
8. *If $Y \subseteq B$, then $A \times Y \subseteq A \times B$.*

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Definition. If A and B are sets, then the **Cartesian product**, $A \times B$, of A and B is the set of all ordered pairs (x, y) where $x \in A$ and $y \in B$. We use the notation $A \times B$ for the Cartesian product of A and B, and using set builder notation, we can write

$$A \times B = \{(x, y) \mid x \in A \text{ and } y \in B\}.$$

We frequently read $A \times B$ as "A cross B." In the case where the two sets are the same, we will write A^2 for $A \times A$. That is,

$$A^2 = A \times A = \{(a, b) \mid a \in A \text{ and } b \in A\}.$$