The goal is to review our high school *set theory* and, using what we have learned thus far in Math 300, make a transition from the highschool concept to the advanced math concepts.

Union and Intersection of two sets

Defs. **Definitions**. Let A_1 and A_2 be subsets of a universal set U. ⟨eg, $U = \mathbb{R}^2$ & draw 2 subsets A_1 and A_2 in $\mathbb{R}^2$⟩ §5.1

1. The **union** of A_1 and A_2, denoted $A_1 \cup A_2$, is the set of all elements that are in A_1 or A_2. Thus p216

$$A_1 \cup A_2 \overset{\text{def}}{=} \{x \in U : x \in A_1 \quad \text{or} \quad x \in A_2\} \overset{\text{other}}{\underset{\text{notation}}{=}} \bigcup_{i=1}^{2} A_i$$

$$\overset{\text{note}}{=} \{x \in U : \text{there exists an } i \in \{1,2\} \text{ such that } x \in A_i\} \overset{\text{other}}{\underset{\text{notation}}{=}} \bigcup_{i \in \{1,2\}} A_i$$

2. The **intersection** of A_1 and A_2, denoted $A_1 \cap A_2$, is the set of all elements in both A_1 and A_2. So

$$A_1 \cap A_2 \overset{\text{def}}{=} \{x \in U : x \in A_1 \quad \text{and} \quad x \in A_2\} \overset{\text{other}}{\underset{\text{notation}}{=}} \bigcap_{i=1}^{2} A_i$$

$$\overset{\text{note}}{=} \{x \in U : \text{for all } i \in \{1,2\} \text{ we have that } x \in A_i\} \overset{\text{other}}{\underset{\text{notation}}{=}} \bigcap_{i \in \{1,2\}} A_i$$

3. The **relative complement of A_1 with respect to A_2**, also called A_2 **set minus** A_1, is the set

$$A_2 \setminus A_1 \overset{\text{def}}{=} \{x \in U : x \in A_2 \quad \text{and} \quad x \notin A_1\} \overset{\text{other}}{\underset{\text{notation}}{=}} A_2 - A_1.$$

4. The **complement** of A_1, denoted $(A_1)^C$, is the set of all elements of U that are not in A_1. So

$$(A_1)^C \overset{\text{def}}{=} \{x \in U : x \notin A_1\} \overset{\text{note}}{=} U \setminus A_1 \overset{\text{note}}{=} U - A_1$$

►. Read the beginning of section 5.1, up to but not including **Set Equality, Susets, and Proper Subsets**. §5.1
After your reading, do the selected problems parts from the variant of ER 5.1.8 (see course HW page). p215–218

Union and Intersection over arbitrary index set

In the above definitions, we had 2 subsets (A_1 and A_2) of a universe U and our index set I was $I = \{1,2\}$. We could make similar definitions for union and intersection of 3 sets (so our indexing set I would be I=$\{1,2,3\}$), or even of 17 sets (so our indexing set I would be $I = \{1,2,\ldots,17\}$). proofed to h

Defs. **Definitions**. ⟨Warning. Do not mix up the universe U and indexing set I; indeed, U and I serve different purposes.⟩ §5.5
Let U be a universe. Let I be a nonempty set ⟨we call I the *index set*⟩. p265
Consider a family ⟨i.e., collection⟩ of subsets $\{A_i : i \in I\}$ of U ⟨so $A_i \subseteq U$ for each $i \in I$⟩.

5. The **union** of family/collection of subsets $\{A_i : i \in I\}$ over the index set I is:

$$\bigcup_{i \in I} A_i \overset{\text{def.}}{=} \{x \in U : \text{there exists an } i \in I \text{ such that } x \in A_i\}$$

$$\overset{\text{i.e.}}{=} \{x \in U : x \text{ is in (at least) one of the } A_i\text{'s}\} \overset{\text{note}}{\subseteq} U.$$

6. The intersection of family/collection of subsets $\{A_i : i \in I\}$ over the index set I is:

$$\bigcap_{i \in I} A_i \overset{\text{def.}}{=} \{x \in U : \text{for all } i \in I \text{ we have that } x \in A_i\}$$

$$\overset{\text{i.e.}}{=} \{x \in U : x \text{ is in all of the } A_i\text{'s}\} \overset{\text{note}}{\subseteq} U.$$

So:.

$$\left[x \in \bigcup_{i \in I} A_i \right] \Leftrightarrow \Big[(\exists i \in I)\ [x \in A_i] \Big] \quad \text{while} \quad \left[x \in \bigcap_{i \in I} A_i \right] \Leftrightarrow \Big[(\forall i \in I)\ [x \in A_i] \Big]$$

►. Read the beginning of section 5.5, up to but not including **Properties of Union and Intersection**. §5.5
We will more so follow the notation in this handout rather than in the book. p264–269
After reading, do the selected problem parts from the variant of ER 5.5.2 and 5.5.3. (see HW page).