▹. The Principle of Mathematical Induction, abbreviated by PMI, is often just called Induction.

Setup. Let $\mathbb{N} = \{1, 2, 3, \dots\}$ be the natural numbers.
Let $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ be the integers.
Let $P(n)$ be an *open sentence* (also called *predicate*) in the variable n, where n is in a subset S of $\mathbb{Z}$.
So when a specified value for $n \in S$ is assigned, $P(n)$ is a statement. Sometimes we denote $P(n)$ by P_n.

For a $n_0 \in \mathbb{N}$, let $\mathbb{N}^{\geq n_0} := \{n \in \mathbb{N} \colon n \geq n_0\} \overset{\text{i.e.}}{=} \{n_0, n_0 + 1, n_0 + 2, n_0 + 3, \dots\} \overset{\text{so}}{\subseteq} \mathbb{N}$.

For a $n_0 \in \mathbb{Z}$, let $\mathbb{Z}^{\geq n_0} := \{n \in \mathbb{Z} \colon n \geq n_0\} \overset{\text{i.e.}}{=} \{n_0, n_0 + 1, n_0 + 2, n_0 + 3, \dots\} \overset{\text{so}}{\subseteq} \mathbb{Z}$.

INDUCTION

1. Induction (basic form) $(\forall n \in \mathbb{N})\,[\,P(n)\,]$ §4.1 p173

If

BASE STEP: $P(1)$ is true

INDUCTIVE STEP: for each $n \in \mathbb{N}$: $\underbrace{[\,P(n) \text{ is true}\,]}_{\text{inductive hypothesis}} \Longrightarrow \underbrace{[\,P(n+1) \text{ is true}\,]}_{\text{inductive conclusion}}$

then $P(n)$ is true for each $n \in \mathbb{N}$.

2. Induction (extended/generalized form) $\left(\forall n \in \mathbb{Z}^{\geq n_0}\right)[\,P(n)\,]$ §4.2 p190

⟨doesn't matter where you start from so let's start at n_0 instead of at 1⟩

Fix $n_0 \in \mathbb{Z}$.
If

BASE STEP: $P(n_0)$ is true

INDUCTIVE STEP: for each $n \in \mathbb{Z}^{\geq n_0}$: $\underbrace{[\,P(n) \text{ is true}\,]}_{\text{inductive hypothesis}} \Longrightarrow \underbrace{[\,P(n+1) \text{ is true}\,]}_{\text{inductive conclusion}}$

then $P(n)$ is true for each $n \in \mathbb{Z}^{\geq n_0}$.

3. Strong Induction (also called complete induction, our book calls this 2^{nd} PMI) §4.2 p194

Fix $n_0 \in \mathbb{Z}$.
If

BASE STEP: $P(n_0)$ is true

INDUCTIVE STEP: for each $n \in \mathbb{Z}^{\geq n_0}$: $\underbrace{[\,P(j) \text{ is true for } j \in \{n_0, 1 + n_0, \dots, n\}\,]}_{\text{inductive hypothesis}} \Rightarrow \underbrace{[\,P(n+1) \text{ is true}\,]}_{\text{inductive conclusion}}$

then $P(n)$ is true for each $n \in \mathbb{Z}^{\geq n_0}$.

Math Induction Writing Guidelines

When writing an induction proof, you need to follow each of the general proof writing guidelines from class. Remember to *keep your reader informed*; thus for an induction proof you need to:

(1) say what you are trying to show inductively
(2) say what your induction variable is (e.g., if you are trying to show $(\forall n \in \mathbb{N})\,[P(n)]$ then say: We will show that *blub* holds for each $n \in \mathbb{N}$ by induction on n.)
(3) indicate where your base step begins
(4) indicate where your base step ends
(5) indicate where your inductive step begins
(6) clearly state your inductive hypothesis (IH)
(7) clearly state your inductive conclusion (IC)
(8) indicate where your inductive step ends.

As with any proof, clean up your *Thinking Land*:

(9) do NOT pull your reader through the mud with you
(10) in the base step confirm only the cases of n (or whatever letter you call you inductive variable) that you need to show (no more and no less).