Defs:. p54

1. A **variable** is a symbol representing an arbitrary (i.e., unspecified, generic) object that can be chosen from a given set U.
2. The set U is called the **universal set for the variable**. So the *universal set for the variable* is the set of specified objects from which objects may be chosen to substitute for the variable.
3. A **constant** is a specific member of the universal

Ex1. So in $x \in \mathbb{R}$, the x is the variable and the universe is $\mathbb{R}$. A constant would be 17.

Defs:. p56/58

4. An **open sentence** is a sentence $P(x_1, x_2, \ldots, x_n)$ involving variables $x_1, x_2, \ldots, x_n$ with the property that when specific values from the universal set are assigned to $x_1, x_2, \ldots, x_n$, the result is a statement (i.e., a declaritive sentence that is either true or false, but not both).
5. BTW: in other classes, you might have he4(3)+5 , ard an open sentence called a **predicate** or a **propositional function**.
6. The **truth set of an open sentence with one variable** is the collection of objects in the universal set that can be substituted for the variable to make the open sentence a true statement.

Ex2. An example of an open sentence $P(x)$ is $x^2 = 9$, with the universe being $\mathbb{R}$.
The truth set for $P(x)$ is $\{3, -3\}$.

▸. Set builder notation takes the form

$$\{x \in U \colon P(x)\}$$

where x is the variable, U is the universal set, and $P(x)$ is the rule/restriction/property that the variable x must satify to be in the set. In Example 2 above, we considered the set

$$\left\{x \in \mathbb{R} \colon x^2 = 9\right\}.$$

Note

$$\underbrace{\left\{x \in \mathbb{R} \colon x^2 = 9\right\}}_{\text{set builder notation}} = \underbrace{\{3, -3\}}_{\text{roster method}} .$$

The roster method just lists the set's elements between curly braces.

Ex3. Some Set Notation for the set $A \overset{\text{def}}{:=} \{5, 9, 13, 17, 21, 25 \ldots\}$

$$\underbrace{\{5, 9, 13, 17, 21, 25 \ldots\}}_{\text{roster method}} \overset{\text{increase}}{\underset{\text{by 4 so}}{=}} \left\{ \overbrace{4(0)+5}^{=5}, \overbrace{4(1)+5}^{=9}, \overbrace{4(2)+5}^{=13}, \overbrace{4(3)+5}^{=17}, \overbrace{4(4)+5}^{=21}, \overbrace{4(5)+5}^{=25}, \ldots \right\}$$

$$= \underbrace{\{4n + 5 \in \mathbb{N} \colon n \in \{0\} \cup \mathbb{N}\}}_{\text{set notation but not set builder notation}}$$

$$= \underbrace{\left\{ x \in \mathbb{N} \colon \overbrace{x = 4n + 5 \text{ for some } n \in \{0\} \cup \mathbb{N}}^{P(x)} \right\}}_{\text{set builder notation}}$$

and if we want to replace $\{0\} \cup \mathbb{N}$ with just $\mathbb{N}$, then what adjustments needed? well ...

$$= \left\{ \overbrace{4(1)+1}^{=5}, \overbrace{4(2)+1}^{=9}, \overbrace{4(3)+1}^{=13}, \overbrace{4(4)+1}^{=17}, \overbrace{4(5)+1}^{=21}, \overbrace{4(6)+1}^{=25}, \ldots \right\}$$

$$= \underbrace{\{4n + 1 \in \mathbb{N} \colon n \in \mathbb{N}\}}_{\text{set notation but not set builder notation}}$$ ⟨think DA: divide a number in A by 4 and will get remainder 1⟩

$$= \underbrace{\left\{ y \in \mathbb{N} \colon \overbrace{y = 4n + 1 \text{ for some } n \in \mathbb{N}}^{P(x)} \right\}}_{\text{set builder notation}}.$$

Ex4. In class.

Recall Some Set Theory/Notation

Defs. Let A and B be subsets of some universal set U.

1. The sets A and B are **equal** when they have precisely the same elements. §2.3 p55

   If A and B are equal, then we write $A = B$. If A and B are not equal, then we write $A \neq B$.

2. The set A is a **subset** B provided that each element of A is an element of B. §2.3 p55

   If A is a subset of B, then we write $A \subseteq B$ and also say A is **contained** in B or say B **contains** A.

   When A is not a subset of B, we write $A \not\subseteq B$.

3. The subset $A \setminus B$ of U is defined by §5.1 p216

$$A \setminus B \stackrel{\text{def}}{=} \{x \in U : x \in A \quad \text{but} \quad x \notin B\}.$$

   The subset $A \setminus B$ is called a **set difference** and is read: A set take away B or A set minus B.

4. When a set contains no elements, we say that the set is the **empty set**. §2.3 p60

   In mathematics, the empty set is usually designated by the symbol $\emptyset$.

   ⟨The symbol $\emptyset$ is the last letter in the Danish-Norwegian alphabet.⟩

Helpful in Proofs

Rmk. $[A \subseteq B] \overset{\text{by def.}}{\iff} [\ x \in A \implies x \in B\ ]$

$[B \subseteq A] \overset{\text{by def.}}{\iff} [\ x \in B \implies x \in A\ ]$

$[A = B] \overset{\text{by def.}}{\iff} [\ x \in A \iff x \in B\ ]$ ... so we get ... $[A = B] \overset{\text{so get}}{\iff} [\ (A \subseteq B) \wedge (B \subseteq A)\ ]$

Ex5. In class.