Review

Def. Two statements $\widetilde{P}$ and $\widetilde{Q}$ are **(logically) equivalent** provided they have the same truth value for each possible combinations of truth values for all the atoms appearing in $\widetilde{P}$ and $\widetilde{Q}$. §2.2 p43
We denote $\widetilde{P}$ is (logically) equivalent to $\widetilde{Q}$ (i.e., $\widetilde{P}$ and $\widetilde{Q}$ are (logically) equivalent) by: $\widetilde{P} \equiv \widetilde{Q}$.
Note, $\equiv$ is used between statements while $=$ is used between numbers.

Def.
- The **converse** of the conditional statement $P \Rightarrow Q$ is the conditional statement $Q \Rightarrow P$. p44
- The **contrapositive** of the conditional statement $P \Rightarrow Q$ is the conditition statement $(\sim Q) \Rightarrow (\sim P)$.
- **Rmk**. We have already seen that: $[P \Rightarrow Q] \not\equiv [Q \Rightarrow P]$ but $[\ P \Rightarrow Q] \equiv [(\sim Q) \Rightarrow (\sim P)\ ]$.

Def. A **negation** (also called **denial**) of a statement P is $\sim P$. p33

Recall. The priority of connectives is: $\sim$ (high, so do first) , $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$ (low, so do last).
So $\sim P \vee \sim Q$ is an abbreviation for $(\sim P) \vee (\sim Q)$.

Important Logical Equivalencies

Theoremm 2.8. Let P, Q, and R be statements. Thm 2.8 §2.2 p48

Double Negation:

$$[\ \sim (\sim P)\ ] \ \equiv \ P\ . \tag{1}$$

Biconditional Statement:

$$[\ P \Leftrightarrow Q\ ] \ \equiv \ [\ (P \Rightarrow Q) \wedge (Q \Rightarrow P)\ ]\ . \tag{2}$$

De Morgans Laws:

$$[\sim (P \wedge Q)\ ] \ \equiv \ [\ (\sim P) \vee (\sim Q)\ ] \tag{3}$$
$$[\ \sim (P \vee Q)\ ] \ \equiv \ [\ (\sim P) \wedge (\sim Q)\ ]\ . \tag{4}$$

Distributive Laws:

$$[\ P \vee (Q \wedge R)\ ] \ \equiv \ [\ (P \vee Q) \wedge (P \vee R)\ ] \tag{5}$$
$$[\ P \wedge (Q \vee R)\ ] \ \equiv \ [\ (P \wedge Q) \vee (P \wedge R)\ ]\ . \tag{6}$$

Conditional Statements:

$$[\ P \Rightarrow Q\ ] \ \equiv \ [\ (\sim Q) \Rightarrow (\sim P)\ ] \quad \text{(contrapositive)} \tag{7}$$
$$[\ P \Rightarrow Q\ ] \ \equiv \ [\ (\sim P) \vee Q\ ] \quad \text{(how do you keep a promise?)} \tag{8}$$
$$[\ \sim (P \Rightarrow Q)\ ] \ \equiv \ [\ P \wedge (\sim Q)\ ] \quad \text{(how do you break a promise?)} \tag{9}$$
$$[\ \sim (P \wedge Q)\ ] \ \equiv \ [\ P \Rightarrow (\sim Q)\ ]\ . \quad \text{(not in book)} \tag{10}$$

Conditionals with Disjunctions:

$$[\ (P \vee Q) \Rightarrow R\ ] \ \equiv \ [\ (P \Rightarrow R) \wedge (Q \Rightarrow R)\ ] \tag{11}$$
$$[P \Rightarrow (Q \vee R)\ ] \ \equiv \ [\ (P \wedge (\sim Q)) \Rightarrow R\ ]\ . \tag{12}$$