1. Definitions/Key Idea

1.1. A **statement** is a declarative sentence that is either true or false but not both; thus, a statement has exactly one **truth value**: true (T) or false (F). §1.1 p1
We often represent a statement by a letter (such as P), called a **statement variable**, similar to the way we represent a number by a variable (such as x).

▷. An example illustrating definitions to come. The compound statement $(\ P \Rightarrow Q\ )\vee(\ R \Leftrightarrow (\sim Q)\ )$ has 3 atoms (namely: P, Q, R) and 4 connectives (namely: $\Rightarrow$, $\vee$, $\Leftrightarrow$, $\sim$).

1.2. A **logical operator** (or **connective**) on statement(s) is a word or combinations of words (e.g.: impplies, and, or, if-then) that combines one or more statements to make a new statement. §2.1 p33

1.3. An **atomic statement** (or **atom**) is a statement satisfying that no part of it is itself a statement. (e.g., P) ∄

1.4. A **compound statement** is a statement that contains one or more connectives. §2.1 p33
A compound statement can be decomposed into its atom(s) and connective(s).

1.5. A **truth table** of a statement exhibits the truth values (TorF) of the statement for each possible combination of truth values for its atoms. §1.1 p6

1.6. A **tautology** is a statement that is true for each assignment of truth values to its atom(s). §2.1 p40

1.7. A **contradiction** is a statement that is false for each assignment of truth values to its atom(s). §2.1 p40

1.8. Two statements $\widetilde{P}$ and $\widetilde{Q}$ are **(logically) equivalent** provided they have the same truth value for each possible combinations of truth values for all the atoms appearing in $\widetilde{P}$ and $\widetilde{Q}$. §2.2 p43
We denote $\widetilde{P}$ is (logically) equivalent to $\widetilde{Q}$ by: $\widetilde{P} \equiv \widetilde{Q}$.
Note, $\equiv$ is used between statements while $=$ is used between numbers.

2. Connective Symbols and Truth Tables

2.1.

	negation
P	∼ P
T	F
F	T

		conjunction and	disjunction or	conditional (implication/if-then)	biconditional (if and only if)
P	Q	P∧ Q	P∨Q	P⇒Q	P⇔Q
T	T	**T**	T	T	T
T	F	F	T	**F**	F
F	T	F	T	T	F
F	F	F	**F**	T	T

2.2. The truth table of a compound statement consisting of n atoms has 2^n lines.
This exhausts all possible truth values of the atoms.
On written work, follow the book's (as done above) pattern of the last listed atomic statement column has alternating T's and F's.

2.3. As in algebra, we give connectives a priority ordering to resolves ambiguities when parentheses are omitted. ∄
The **priority/precedence of the connectives** is:

$\sim$ (high, so do first) , $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$ (low, so do last) .

Ex. $P \implies \sim Q \vee R \Leftrightarrow S$ is an abbreviation for $(\ P \implies [\ (\ \sim Q) \vee R\ ]\ ) \Leftrightarrow S$.

2.4. Use $P \Rightarrow Q$ to translate: (Good check if you have grasp of 2.4 is the book's ER 2.1.13 p. 42).) §2.1 p37

(1) If P, then Q

(2) P implies Q

(3) P is sufficient for Q

(4) P only if Q

(5) P only when Q

(6) Q is necessary for P

(7) Q if P

(8) Q when P (also Q whenever P)

Rmk on (4). What sounds correct to your ear?

- $x > 0$ only if $x > 17$.
- $x > 17$ only if $x > 0$.

2.5. Use $P \Leftrightarrow Q$ to translate: §2.1 p39

(1) P is equivalent to Q

(2) P if and only if Q (See above $P \Rightarrow Q$'s (4) and (7))

(3) P if but only if Q

(4) P precisely when Q

(5) P is necessary and sufficient for Q (See above $P \Rightarrow Q$'s, (3) and (6))