Defs:. p54

1. A **variable** is a symbol representing an arbitrary (i.e., unspecified, generic) object that can be chosen from a given set U.
2. The set U is called the **universal set for the variable**. So the *universal set for the variable* is the set of specified objects from which objects may be chosen to substitute for the variable.
3. A **constant** is a specific member of the universal

Ex1. So in $x \in \mathbb{R}$, the x is the variable and the universe is $\mathbb{R}$. A constant would be 17.

Defs:. p56/58

4. An **open sentence** is a sentence $P(x_1, x_2, \ldots, x_n)$ involving variables $x_1, x_2, \ldots, x_n$ with the property that when specific values from the universal set are assigned to $x_1, x_2, \ldots, x_n$, the result is a statement (i.e., a declaritive sentence that is either true or false, but not both).
5. An open sentence is also called a **predicate** or a **propositional function**.
6. The **truth set of an open sentence with one variable** is the collection of objects in the universal set that can be substituted for the variable to make the open sentence a true statement.

Ex2. An example of an open sentence $P(x)$ is $x^2 = 9$, with the universe being $\mathbb{R}$.
The truth set for $P(x)$ is $\{3, -3\}$.

▸. Set builder notation takes the form

$$\{x \in U \colon P(x)\}$$

where x is the variable, U is the universal set, and $P(x)$ is the rule/restriction that x must satify to be in the set. In Example 2 above, we considered the set

$$\{x \in \mathbb{R} \colon x^2 = 9\}.$$

Note

$$\{x \in \mathbb{R} \colon x^2 = 9\} = \{3, -3\}$$

where the left side is in set builder notation while the right side uses the roster method.
The roster method just lists the set's elements between braces.

Ex3. Some Set Notation

$$\underbrace{\{1, 4, 7, 10, 13, \ldots\}}_{\text{roster method}} \overset{\text{think}}{\underset{\text{D.A.}}{=}} \{\, 3(1)-2\ ,\ 3(2)-2\ ,\ 3(3)-2\ ,\ 3(4)-2\ ,\ \ldots \}$$

$$= \underbrace{\left\{ x \in \mathbb{N} \colon \overbrace{x = 3n - 2 \text{ for some } n \in \mathbb{N}}^{P(x)} \right\}}_{\text{set builder notation}}$$

$$= \underbrace{\{3n - 2 \in \mathbb{N} \colon n \in \mathbb{N}\}}_{\text{set notation but not set builder notation}}$$

$$\overset{\text{or}}{=} \underbrace{\{3n + 1 \in \mathbb{N} \colon n \in \mathbb{N} \cup \{0\}\}}_{\text{set notation but not set builder notation}}$$

Ex4. In class.

Def. When a set contains no elements, we say that the set is the **empty set**. p60
In mathematics, the empty set is usually designated by the symbol $\emptyset$.
⟨The symbol $\emptyset$ is the last letter in the Danish-Norwegian alphabet and is LaTex-ed \emptyset.⟩

Defs:. Let A and B be two sets. p55

7. The sets A and B are **equal** when they have precisely the same elements.
If A and B are equal, then we write $A = B$. If A and B are not equal, then we write $A \neq B$.
8. The set A is a **subset** B provided that each element of A is an element of B.
If A is a subset of B, then we write $A \subseteq B$ and also say A is **contained** in B or say B **contains** A.
When A is not a subset of B, we write $A \not\subseteq B$.

Rmk. $[A \subseteq B] \overset{\text{by def.}}{\iff} [\, x \in A \implies x \in B \,]$
$[B \subseteq A] \overset{\text{by def.}}{\iff} [\, x \in B \implies x \in A \,]$
$[A = B] \overset{\text{by def.}}{\iff} [\, x \in A \iff x \in B \,]$ ... so we get ... $[A = B] \overset{\text{so get}}{\iff} [\, (A \subseteq B) \wedge (B \subseteq A) \,]$

Ex5. In class.