Definitions

Def. Two statements $\widetilde{P}$ and $\widetilde{Q}$ are **logically equivalent** provided they have the same truth value for all possible combinations of truth values for all variables (i.e. all atoms) appearing in $\widetilde{P}$ and $\widetilde{Q}$. In this case, we write $\widetilde{P} \equiv \widetilde{Q}$ and say that $\widetilde{P}$ and $\widetilde{Q}$ are logically equivalent. p43

Def.
- The **converse** of the conditional statement $P \Rightarrow Q$ is the conditional statement $Q \Rightarrow P$. p44
- The **contrapositive** of the conditional statement $P \Rightarrow Q$ is the condition statement $(\sim Q) \Rightarrow (\sim P)$.
- **Rmk**. We have already seen that: $[P \Rightarrow Q] \not\equiv [Q \Rightarrow P]$ but $[\, P \Rightarrow Q] \equiv [(\sim Q) \Rightarrow (\sim P)\,]$.

Def. A **negation** (also called **denial**) of a statement P is $\sim P$. p33

Recall. The priority of connectives is: $\sim$ (high) , $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$ (low). So $\sim P \vee \sim Q$ is $(\sim P) \vee (\sim Q)$.

Important Logical Equivalencies

Theoremm 2.8. Let P, Q, and R be statements. Thm 2.8 p48

Double Negation:

$$[\,\sim (\sim P)\,] \;\equiv\; P\,. \tag{1}$$

Biconditional Statement:

$$[\,P \Leftrightarrow Q\,] \;\equiv\; [\,(P \Rightarrow Q) \wedge (Q \Rightarrow P)\,]\,. \tag{2}$$

De Morgans Laws:

$$[\sim (P \wedge Q)\,] \;\equiv\; [\,(\sim P) \vee (\sim Q)\,] \tag{3}$$

$$[\,\sim (P \vee Q)\,] \;\equiv\; [\,(\sim P) \wedge (\sim Q)\,]\,. \tag{4}$$

Distributive Laws:

$$[\,P \vee (Q \wedge R)\,] \;\equiv\; [\,(P \vee Q) \wedge (P \vee R)\,] \tag{5}$$

$$[\,P \wedge (Q \vee R)\,] \;\equiv\; [\,(P \wedge Q) \vee (P \wedge R)\,]\,. \tag{6}$$

Conditional Statements:

$$[\,P \Rightarrow Q\,] \;\equiv\; [\,(\sim Q) \Rightarrow (\sim P)\,] \qquad \text{(contrapositive)} \tag{7}$$

$$[\,P \Rightarrow Q\,] \;\equiv\; [\,(\sim P) \vee Q\,] \qquad \text{(how do you keep a promise?)} \tag{8}$$

$$[\,\sim (P \Rightarrow Q)\,] \;\equiv\; [\,P \wedge (\sim Q)\,] \qquad \text{(how do you break a promise?)} \tag{9}$$

$$[\,\sim (P \wedge Q)\,] \;\equiv\; [\,P \Rightarrow (\sim Q)\,]\,. \qquad \text{(not in book)} \tag{10}$$

Conditionals with Disjunctions:

$$[\,(P \vee Q) \Rightarrow R\,] \;\equiv\; [\,(P \Rightarrow R) \wedge (Q \Rightarrow R)\,] \tag{11}$$

$$[P \Rightarrow (Q \vee R)\,] \;\equiv\; [\,(P \wedge (\sim Q)) \Rightarrow R\,]\,. \tag{12}$$