An *Evaluation of Proofs* Exercise

▷. **Conjecture B**. For all real numbers x and y, if $x \neq y$, $x > 0$, and $y > 0$, then

$$\frac{x}{y} + \frac{y}{x} > 2.$$

Proposed Proof. Since x and y are strictly positive real numbers, xy is strictly positive and we can multiply both sides of the inequality by xy to obtain

$$\begin{aligned} \left(\frac{x}{y} + \frac{y}{x}\right) \cdot xy &> 2 \cdot xy \\ x^2 + y^2 &> 2xy. \end{aligned}$$

By combining all terms on the left side of the inequality, we see that $x^2 - 2xy + y^2 > 0$ and then by factoring the left side, we obtain $(x - y)^2 > 0$. Since $x \neq y$, $(x - y) \neq 0$ and so $(x - y)^2 > 0$. This proves that if $x \neq y$, $x > 0$, and $y > 0$, then $\frac{x}{y} + \frac{y}{x} > 2$. □

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Optional Thinking Land Space

Symbolically:

$$(\forall x \in \mathbb{R})\ (\forall y \in \mathbb{R}) \left[(x \neq y \ \wedge \ x > 0 \ \wedge \ y > 0) \implies \frac{x}{y} + \frac{y}{x} > 2 \right].$$