Writing Guidelines

The below *Guidelines for Writing Mathematical Proofs* are collected in Appendix A (p. 492–496) of Mathematical Reasoning. Writing and Proof (Version 3) by Ted Sundstrom, located at

`https://people.math.sc.edu/girardi/m300/book/Sundstrom3.pdf` .

These 15 guidelines are introduced within sections: 1.1, 1.2, 3.1, 3,3, 4.1.

▹ A ▹ indicates additional comments by Prof. Girardi.

1. Know your audience. do first §3.1 #94 p1

Every writer should have a clear idea of the intended audience for a piece of writing. In that way, the writer can give the right amount of information at the proper level of sophistication to communicate effectively. This is especially true for mathematical writing. For example, if a mathematician is writing a solution to a textbook problem for a solutions manual for instructors, the writing would be brief with many details omitted. However, if the writing was for a students solution manual, more details would be included.

▹ Our intended audience is a fellow student in a USC regular section of this course.

2. Begin with a carefully worded statement of the theorem or result to be proven. §1.2 #1 p22

The statement should be a simple declarative statement of the problem. Do not simply rewrite the problem as stated in the textbook or given on a handout. Problems often begin with phrases such as "Show that" or "Prove that". This should be reworded as a simple declarative statement of the theorem. Then skip a line and write "Proof" in italics or boldface font (when using a word processor). Begin the proof on the same line. Make sure that all paragraphs can be easily identified. Skipping a line between paragraphs or indenting each paragraph can accomplish this.

As an example, an exercise in a text might read, "Prove that if x is an odd integer, then x^2 is an odd integer." This could be started as follows:

Theorem. If x is an odd integer, then x^2 is an odd integer.

Proof. Let x be an odd integer. We shall show that x^2 is an odd integer.
Since x is an odd integer, there is $n \in \mathbb{Z}$ such that ... □

Incorrect Proof. We shall show that if x is an odd integers, then x^2 is odd.
Since x is an odd integer, there is $n \in \mathbb{Z}$ such that ... □

The mistake in the incorrect proof is that the proof starts by saying if x is odd but in the next paragraph the proof says since x is odd (before declaring that x is *actually* odd).

3. Begin the proof with a statement of your assumptions. §1.2 #2 p23

Follow the statement of your assumptions with a statement of what you will prove.

Theorem. The product of two odd integers is an odd integer.

Proof. Let x and y be odd integers. We will show that xy is an odd integer.
Since x is an odd integer, there is $n \in \mathbb{Z}$ such that ... □

▹ When starting a direct proof (the type of proofs in Chapters 1 and 2), the wording *Let x be* ⟨similar to above⟩ is preferred over the the wording *We assume that x is* ⟨as book often uses⟩. Basically, *let* is better in direct proofs (as in Chs. 1 and 2) while *assume* is better for proof by contradiction (in Ch. 3).

4. Use the pronoun "we". §1.2 #3 p23

If a pronoun is used in a proof, the usual convention is to use "we" instead of "I." The idea is to stress that you and the reader are doing the mathematics together. It will help encourage the reader to continue working through the mathematics. Notice that we started the proof of Theorem 1.8 with "We assume that ... ".

5. Use italics for variables when using a word processor. §1.2 #4 p23

When using a word processor to write mathematics, the word processor needs to be capable of producing the appropriate mathematical symbols and equations. The mathematics that is written with a word processor should look like typeset mathematics. This means that variables need to be italicized, boldface is used for vectors, and regular font is used for mathematical terms such as the names of the trigonometric functions and logarithmic functions.

For example, we do not write sin x or *sin x*. The proper way to typeset this is $\sin x$.

6. Do not use $*$ for multiplication or ^ for exponents. §3.1 #5 p95

Leave this type of notation for writing computer code. The use of this notation makes it difficult for humans to read. In addition, avoid using / for division when using a complex fraction.

For example, it is very difficult to read $(x^3 - 3x^2 + 1/2)/(2x/3 - 7)$; the fraction

$$\frac{x^3 - 3x^2 + \frac{1}{2}}{\frac{2x}{3} - 7}$$

is much easier to read.

7. Use complete sentences and proper paragraph structure. §3.1 #2 p94

Good grammar is an important part of any writing. Therefore, conform to the accepted rules of grammar. Pay careful attention to the structure of sentences. Write proofs using **complete sentences** but avoid run-on sentences. Also, do not forget punctuation, and always use a spell checker when using a word processor.

8. Keep the reader informed. §3.3 #– p119 and §4.1 #– p176

Sometimes a theorem is proven by proving the contrapositive or by using a proof by contradiction. If either proof method is used, this should be indicated within the first few lines of the proof. This also applies if the result is going to be proven using mathematical induction. Examples:

- We will prove this result by proving the contrapositive of the statement.
- We will prove this statement using a proof by contradiction.
- We will assume to the contrary that ...
- We will use mathematical induction to prove this result.

In addition, make sure the reader knows the status of every assertion that is made. That is, make sure it is clearly stated whether an assertion is an assumption of the theorem, a previously proven result, a well-known result, or something from the readers mathematical background.

9. Display important equations and mathematical expressions. §1.2 #5 p23

Equations and manipulations are often an integral part of the exposition. Do not write equations, algebraic manipulations, or formulas in one column with reasons given in another column (as is often done in geometry texts). Important equations and manipulations should be displayed. This means that they should be centered with blank lines before and after the equation or manipulations, and if one side of an equation does not change, it should not be repeated. For example,

Using algebra, we obtain

$$\begin{aligned} x \cdot y &= (2m+1)(2n+1) \\ &= 4mn + 2m + 2n + 1 \\ &= 2(2mn + m + n) + 1. \end{aligned}$$

Since m and n are integers, we conclude that ... .

10. Equation numbering guidelines. §1.2 #5 p23

If it is necessary to refer to an equation later in a proof, that equation should be centered and displayed, and it should be given a number. The number for the equation should be written in parentheses on the same line as the equation at the right-hand margin.

Example:
Since x is an odd integer, there exists an integer n such that

$$x = 2n + 1. \tag{1}$$

Later in the proof, there may be a line such as

Then, using the result in equation (1), we obtain . . . .

Please note that we should only number those equations we will be referring to later in the proof. Also, note that the word "equation" is not capitalized when we are referring to an equation by number. Although it may be appropriate to use a capital "E," the usual convention in mathematics is not to capitalize.

11. Do not use a mathematical symbol at the beginning of a sentence. §3.1 #6 p95
For example, we should not write, "Let n be an integer. n is an odd integer provided that . . . ." Many people find this hard to read and often have to reread it to understand it. It would be better to write, "An integer n is an odd integer provided that . . . ."

▹ Having some go-to *filler words* handy is helpful.

12. Use English and minimize the use of cumbersome notation. §3.1 #7 p95
Do not use the special symbols for quantifiers $\forall$ (for all), $\exists$ (there exists), $\ni$ (such that), or $\therefore$ (therefore) in formal mathematical writing. It is often easier to write, and usually easier to read, if the English words are used instead of the symbols. For example, why make the reader interpret

$$(\forall x \in \mathbb{R})\ (\exists y \in \mathbb{R})\ [x + y = 0]$$

when it is possible to write

For each real number x, there exists a real number y such that $x + y = 0$,

or more succinctly (if appropriate)

Every real number has an additive inverse.

13. Tell the reader when the proof has been completed. §1.2 #6 p23
Perhaps the best way to do this is to say outright that, "This completes the proof." Although it may seem repetitive, a good alternative is to finish a proof with a sentence that states precisely what has been proven. In any case, it is usually good practice to use some "end of proof symbol" such as ■.

14. Keep it simple. §3.1 #3 p94
It is often difficult to understand a mathematical argument no matter how well it is written. Do not let your writing help make it more difficult for the reader. Use simple, declarative sentences and short paragraphs, each with a simple point.

15. Write a first draft of your proof and then revise it. §3.1 #4 p95
Remember that a proof is written so that readers are able to read and understand the reasoning in the proof. Be clear and concise. Include details but do not ramble. Do not be satisfied with the first draft of a proof. Read it over and refine it. Just like any worthwhile activity, learning to write mathematics well takes practice and hard work. This can be frustrating. Everyone can be sure that there will be some proofs that are difficult to construct, but remember that proofs are a very important part of mathematics. So work hard and have fun.