These *Practice Problems* are a sampling of the type of problems which could be an exam.
These problem are, in no way, meant as a comprehensive review for the exam.

Math is not a spectator sport.
Often we learn more from our failed attempts at a proof rather than reading a clean proof.
So give these problems a solid attempt before seeking help, e.g.:
looking through your notes and/or book, looking at the posted hints, or looking at Piazza.
Some hints are very generous. Do not except such generous hints on the exam.

You are highly encouraged to discuss and work together on these problems.
Our course Piazza page should help with this (let's use Piazza's `exam` folder).

- Post a note on Piazza saying, e.g., Fred has reserved TCL Study Room 123 for this Monday (7/4) for 7–9pm. Let me know if you want to join. Link to reserve a TCL Study Room on-line.
- Since the problems are not to be handed in (but rather serve as practice for exams), you may be sharing and posting not only ideas but also solutions. In Piazza (look at horizontal bar near top), the folder `piazza` contains a note about inserting a (PDF/JPEG) file.

▷. If divides is not in the material covered on the exam, then the following definition will be given on the exam.
Def. A nonzero integer n **divides** an integer b, denoted $n|b$, provided that $(\exists k \in \mathbb{Z})\ [nk = b]$.

▷. Recall the definition. A **counterexample** to a statement of the form $(\forall x \in U)\ [P(x)]$ is an object c in the universal set U for which $P(c)$ is false. So a counterexample is an example that shows $(\forall x \in U)\ [P(x)]$ is false.

The Practice Problems

★★★. **Instructions**. Below are several conjectures.

- If the conjecture is true, give a proof of the conjecture.
- If the conjecture is false, then give a counterexample to the conjecture. Explain why your counterexample does indeed show that the conjecture is false.

Conjecture A. For each natural number x strictly greater than 1, the quantity $x^2 + x + 41$ is prime.
Conjecture B. For each real number x, we have that $x + y = 0$ for some real number y.
Conjecture C. $(\forall x \in \mathbb{R})\ (\forall y \in \mathbb{R})\ [\ (x > 1 \wedge y > 0) \implies y^x > x]$
Conjecture D. For integers a, b, and c with $a \neq 0$, if a divides bc, then a divides b or a divides c.
Conjecture E. For integers a, b, c, and d with $a \neq 0$, if a divides $b-c$ and a divides $c-d$, then $a \mid (b-d)$.
Conjecture F. For each positive real number x, the inequality $x^2 - x \geq 0$ holds.
Conjecture G. For all positive real numbers x, we have $2^x > x + 1$.
Conjecture H. For every positive real number x, there is a positive real number y less than x with the property that for all positive real numbers z, we have $yz \geq z$.
Conjecture I. For every positive real number x, there is a positive real number y with the property that if $y < x$, then for all positive real numbers z, it holds that $yz \geq z$.

From: *A Transition To Advanced Mathematics* by Smith, Eggen, St. Andre. (6^{th}Ed.1.6.5/p54 = 7^{th}Ed.1.6.4/p57), which is over \$300.

Hints

◦. When writing a proof, follow the proof Writing Guidelines while when writing symbolically, follow the Symbolically Write Guidelines. We have been using these guidelines throughout class.

◦. Avoid common mistakes.

- "x is a positive number" provided $x > 0$ while "x is a nonnegative number" provided $x \geq 0$
- "x is less than y" provided $x < y$ while "x is less than or equal to y" provided $x \leq y$
- don't forget needed parentheses, e.g., $a|b-1$ does not make sense and should be written as $a|(b-1)$.

True/False solutions for the original given statements

(A) false	(D) false	(G) false
(B) true	(E) true	(H) false
(C) false	(F) false	(I) true

Helpful Logical Equivalences.

When negating quantified statements, we often use the logical equivalencies (from our §2.2 Handout):

$$[\sim(P \Rightarrow Q)] \equiv [P \wedge (\sim Q)]$$ (how do you break a promise?)

$$[\sim(P \wedge Q)] \equiv [P \Rightarrow (\sim Q)]$$ (not in book)

$$[\sim(P \wedge Q)] \equiv [(\sim P) \vee (\sim Q)]$$ (De Morgans Law)

$$[\sim(P \vee Q)] \equiv [(\sim P) \wedge (\sim Q)]$$ (De Morgans Law) .

You should have a working knowledge of the logical equivalencies from this handout.

Samples of how the problems might look on an exam.

A. **Conjecture A**. For each natural number x strictly greater than 1, the quantity $x^2 + x + 41$ is prime.

A1. Complete the following two definitions.

- **Def**. A natural number p is a prime number provided ...

  $p \neq 1$ and the only natural numbers that are factors of p are: 1 and p.

- **Def**. A natural number c is a composite number provided ...

  $c \neq 1$ and c is not a prime number

A2. Symbolically write Conjecture A. As the universe, use $\mathbb{N}^{>1}$

$$\left(\forall x \in \mathbb{N}^{>1}\right) \left[x^2 + x + 41 \text{ is prime}\right]$$

A3. Symbolically write an useful negation (i.e., denial) of Conjecture A, without using $\sim$ nor $\neg$.

$$\left(\exists x \in \mathbb{N}^{>1}\right) \left[x^2 + x + 41 \text{ is not prime}\right]$$

Also fine since the universe is $\mathbb{N}^{>1}$ so $x \neq 1$

$$\left(\exists x \in \mathbb{N}^{>1}\right) \left[x^2 + x + 41 \text{ is composite}\right]$$

A4. Is Conjecture A true or false? (circle one) TRUE $\boxed{\text{FALSE}}$.

A5. Is the negation of Conjecture A true or false? (circle one) $\boxed{\text{TRUE}}$ FALSE .

A6. Provide a proof or a counterexample to Conjecture A.

Counterexample. Conjecture A is false, as shown by the counterexample $x_0 = 41$. Note $x_0 = 41 \in \mathbb{N}^{>1}$. For $x_0 = 41$, we have

$$\begin{aligned} x_0^2 + x_0 + 41 &= (41)^2 + 41 + 41 \\ &= 41\,(41 + 1 + 1) \\ &= 41\,(43)\,. \end{aligned}$$

Note the quantity $(41)\,(43)$ is not prime.

Thus, when $x_0 = 41$, we have $x_0 \in \mathbb{N}^{>1}$ but $x_0^2 + x_0 + 41$ is not prime.

B. **Conjecture B**. For each real number x, we have that $x + y = 0$ for some real number y.

B1. Symbolically write Conjecture B.

$$\boxed{(\forall x \in \mathbb{R})\ (\exists y \in \mathbb{R})\ [x + y = 0]}$$

B2. Symbolically write an useful negation of Conjecture B, without using $\sim$ nor $\neg$.

$$\boxed{(\exists x \in \mathbb{R})\ (\forall y \in \mathbb{R})\ [x + y \neq 0]}$$

B3. Is Conjecture B true or false? (circle one) $\boxed{\text{TRUE}}$ FALSE .

B4. Is the negation of Conjecture B true or false? (circle one) TRUE $\boxed{\text{FALSE}}$.

B5. Provide a proof or a counterexample to Conjecture B.

Thinking Land. Fix $x \in \mathbb{R}$. Choose $y = -x$. So $y \in \mathbb{R}$. Then $x + y = x + (-x) = 0$.

Proof of Conjecture B. Towards showing Conjecture B is true, fix $x \in \mathbb{R}$. We will show that there exists $y \in \mathbb{R}$ such that $x + y = 0$.

Let $y = -x$. Then $y \in \mathbb{R}$ and $x + y = x + (-x) = 0$.

We have just shown that Conjecture B is true. □

C. **Conjecture C**. $(\forall x \in \mathbb{R})\ (\forall y \in \mathbb{R})\ [\ (x > 1 \wedge y > 0) \implies y^x > x]$.

C1. Provide an equivalent symbolicaly written expression of Conjecture C by filling in the below line.

$$\left(\forall x \in \mathbb{R}^{>1}\right)\ \left(\forall b \in \mathbb{R}^{>0}\right)\ [\ \underline{\qquad b^x > x \qquad}\ ]\,.$$

C2. Provide an equivalent symbolicaly written expression of Conjecture C by filling in the below line.

$$\left(\forall b \in \mathbb{R}^{>0}\right)\ \left(\forall x \in \mathbb{R}^{>1}\right)\ [\ \underline{\qquad b^x > x \qquad}\ ]\,.$$

⚠. Recall, we can interchange two like quantifier (i.e., two $\forall$ together or two $\exists$ together) but we can NOT interchange two mixed quantifers (i.e., one $\forall$ and one $\exists$).

Hint. Think of b as the base of the expontential function.

C3. Is Conjecture C true or false? (circle one) TRUE $\boxed{\text{FALSE}}$.

C4. Is the negation of Conjecture C true or false? (circle one) $\boxed{\text{TRUE}}$ FALSE .

C5. Provide a proof or a counterexample to Conjecture C.

Thinking Land. Fix $b \in \mathbb{R}^{>0}$.

Consider the function $l\colon (1,\infty) \to \mathbb{R}$ given by $l(x) = x$ ⟨it's graph is a line⟩.

Consider the function $f\colon (1,\infty) \to \mathbb{R}$ given by $f(x) = b^x$.

The function $y = f(x)$ is an expontential function with base $b > 0$.

If $0 < b < 1$, then $y = f(x)$ is decreasing and represents expontential decay.

If $b = 1$, then $y = f(x)$ is the horizontal line $y = 1$

If $1 < b$, then $y = f(x)$ is increasing and represents expontential growth.

Let's try the *easiest* situation first and try $b = 1$. In the orginial statement, b is the y, so try $y = 1$.

Counterexample. Conjecture C is false, as shown by the counterexample $x_0 = 17$ and $y_0 = 1$. Note $x_0, y_0 \in \mathbb{R}$. Also $x_0 = 17 > 1$ and $y_0 = 1 > 0$. However, $(y_0)^{x_0} \not> x_0$ since

$$(y_0)^{x_0} \;=\; 1^{17} = 1 \not> 17 = x_0 \;.$$

Side Remark: When $y_0 = 1$, the x_0 can be any real number strictly larger than 1.

D. **Conjecture D**. For integers a, b, and c with $a \neq 0$, if a divides bc, then a divides b or a divides c.

D1. If divides is not in the material covered on the exam, then the following definition will be given on the exam.

Def. A nonzero integer n **divides** an integer b, denoted $n|b$, provided that $(\exists k \in \mathbb{Z})\ [nk = b]$.

Otherwise, complete the below definition symbolically (not in English).

Def. A nonzero integer n **divides** an integer b, denoted $n|b$, provided that $\underline{\ (\exists k \in \mathbb{Z})\ [nk = b]\ }$.

D2. Symbolically write Conjecture D. As the universe, use $\mathbb{R}^{\neq 0} \times \mathbb{R} \times \mathbb{R}$.

$$\boxed{\left(\forall\, (a,b,c) \in \mathbb{R}^{\neq 0} \times \mathbb{R} \times \mathbb{R}\right)\ [\ a|\,(bc) \implies (a|b \vee a|c)\ ]}$$

D3. Symbolically write an useful negation (i.e., denial) of Conjecture D, without using $\sim$ nor $\neg$.

$$\boxed{\left(\exists\, (a,b,c) \in \mathbb{R}^{\neq 0} \times \mathbb{R} \times \mathbb{R}\right)\ [\ a|\,(bc) \ \wedge\ a \nmid b \ \wedge\ a \nmid c\ ]}$$

Remark: just used $[\ \sim (P \Rightarrow Q)\ ] \equiv [\ P \wedge (\sim Q)\ ]$ ⟨how do you break a promise?⟩

D4. Is Conjecture D true or false? (circle one) TRUE $\boxed{\text{FALSE}}$.

D5. Is the negation of Conjecture D true or false? (circle one) $\boxed{\text{TRUE}}$ FALSE.

D6. Provide a proof or a counterexample to Conjecture D.

Thinking Land. Pick an a that is a product of two primes, say $a = p_1 \cdot p_2$. Then let $b = p_1$ and $c = p_2$.

Counterexample. Conjecture D is false, as shown by the counterexample

$$a = 6 \text{ with } b = 2 \text{ and } c = 3.$$

First note a, b, and c are integers with $a \neq 0$. Note $6|6$ since $6 \cdot 1 = 6$ and $1 \in \mathbb{Z}$. Thus $a|\,(bc)$ since $a = 6$ and $bc = 2 \cdot 3 = 6$. Note $6 \nmid 2$ by def. of divides since if $6k = 2$ then $k = \frac{1}{3} \notin \mathbb{Z}$. Thus $a \nmid b$. Similarly, $6 \nmid 3$ by def. of divides since if $6k = 3$ then $k = \frac{1}{2} \notin \mathbb{Z}$. Thus $a \nmid c$.

E. **Conjecture E**. For integers a, b, c, and d with $a \neq 0$, if a divides $b - c$ and a divides $c - d$, then $a \mid (b - d)$.

E1. If divides is not in the material covered on the exam, then the following definition will be given on the exam.

Def. A nonzero integer n **divides** an integer b, denoted $n|b$, provided that $(\exists k \in \mathbb{Z})\ [nk = b]$.

Otherwise, complete the below definition symbolically (not in English).

Def. A nonzero integer n **divides** an integer b, denoted $n|b$, provided that $\underline{\ (\exists k \in \mathbb{Z})\ [nk = b]\ }$.

E2. Symbolically write Conjecture E. As universe(s), use $\mathbb{Z}$, $\mathbb{Z}^{\neq 0}$ (also can be written $\mathbb{Z} \setminus \{0\}$), and/or some cross products of these.

$$\boxed{\left(\forall a \in \mathbb{Z}^{\neq 0}\right)\ (\forall b \in \mathbb{Z})\ (\forall c \in \mathbb{Z})\ (\forall d \in \mathbb{Z})\ [\ \{a|\,(b-c) \wedge a|\,(c-d)\} \implies a|\,(b-d)]}$$

E3. Symbolically write an useful negation of Conjecture E, without using $\sim$ nor $\neg$.

$$\boxed{\left(\exists a \in \mathbb{Z}^{\neq 0}\right)\ (\exists b \in \mathbb{Z})\ (\exists c \in \mathbb{Z})\ (\exists d \in \mathbb{Z})\ [\ (a|\,(b-c) \wedge a|\,(c-d)) \wedge a \nmid (b-d)]}$$

Remark: just used $[\ \sim (P \Rightarrow Q)\ ] \equiv [\ P \wedge (\sim Q)\ ]$ ⟨how do you break a promise?⟩

E4. Is Conjecture E true or false? (circle one) $\boxed{\text{TRUE}}$ FALSE .

E5. Is the negation of Conjecture E true or false? (circle one) TRUE $\boxed{\text{FALSE}}$.

E6. Provide a proof or a counterexample to Conjecture E.

Thinking Land. Make sure to use *different k's,* e.g., k_1 and k_2. Note by adding the two equalities

$$\begin{aligned} ak_1 &= b - c \\ ak_2 &= c - d \end{aligned}$$

we get

$$a\,(k_1 + k_2) = b - d\ .$$

Proof of Conjecture E. Towards showing Conjecture E is true, let a, b, c, $d \in \mathbb{Z}$ with $a \neq 0$ such that $a|\,(b-c)$ and $a|\,(c-d)$. We will show that $a|\,(b-d)$.

Since $a|\,(b-c)$ and $a|\,(c-d)$ there exists $k_1, k_2 \in \mathbb{Z}$ such that

$$\begin{aligned} ak_1 &= b - c \\ ak_2 &= c - d \end{aligned} \tag{1}$$

Adding the two equations in (1) gives

$$a\,(k_1 + k_2) = b - d\ . \tag{2}$$

Let $j = k_1 + k_2$. Then $j \in \mathbb{Z}$ since $k_1, k_2 \in \mathbb{Z}$ and $\mathbb{Z}$ is closed (under addition). Thus in (2) we wrote $aj = b - d$ for some $j \in \mathbb{Z}$. Thus by definition of divides, $a|\,(b-d)$.

This completes the proof. ∎

F. **Conjecture F**. For each positive real number x, the inequality $x^2 - x \geq 0$ holds.

F1. Symbolically write Conjecture F. As the universe use $\mathbb{R}^{>0}$.

$$\boxed{\left(\forall x \in \mathbb{R}^{>0}\right)\ \left[x^2 - x \geq 0\right]}$$

F2. Symbolically write an useful negation of Conjecture F, without using $\sim$ nor $\neg$.

$$\boxed{\left(\exists x \in \mathbb{R}^{>0}\right)\ \left[x^2 - x < 0\right]}$$

F3. Is Conjecture F true or false? (circle one) TRUE $\boxed{\text{FALSE}}$.

F4. Is the negation of Conjecture F true or false? (circle one) $\boxed{\text{TRUE}}$ FALSE .

F5. Provide a proof or a counterexample to Conjecture F.

Thinking Land. Consider $f(x) = x^2 - x$. So we want to find an x_0 such that $f(x_0) < 0$. Note $f(x) = (x-0)\,(x-1)$ is a concave up parabola that crosses the x-axis when $x = 0$ and $x = 1$ and has vertex at $\left(\frac{1}{2}, f\left(\frac{1}{2}\right)\right) = \left(\frac{1}{2}, -\frac{1}{2}\right)$. So if $x \in (0,1)$ the $f(x) < 0$. So any x in the interval $(0,1)$ would be a counterexample. So just pick any such $x \in (0,1)$ ... make your life easy ... pick an x for which the needed arithmetic $\langle$for $x^2 - x\rangle$ is not too bad.

Counterexample. Conjecture F is false, as shown by the counterexample $x_0 = \frac{1}{2}$. Note x_0 is a positive real number. But

$$(x_0)^2 - x_0 \;=\; \left(\frac{1}{2}\right)^2 - \frac{1}{2} \;=\; \frac{1}{4} - \frac{1}{2} \;=\; -\frac{1}{4} \ngeq 0\,.$$

G. Conjecture G. For all positive real numbers x, we have $2^x > x + 1$.

G1. Symbolically write Conjecture G.

$$\boxed{\left(\forall x \in \mathbb{R}^{>0}\right)\ \left[2^x > x + 1\right]}$$

G2. Symbolically write an useful negation of Conjecture G, without using $\sim$ nor $\neg$.

$$\boxed{\left(\exists x \in \mathbb{R}^{>0}\right)\ \left[2^x \leq x + 1\right]}$$

G3. Is Conjecture G true or false? (circle one) TRUE $\boxed{\text{FALSE}}$.

G4. Is the negation of Conjecture G true or false? (circle one) $\boxed{\text{TRUE}}$ FALSE .

G5. Provide a proof or a counterexample to Conjecture G.

Thinking Land. Graph the exponential function $f(x) = 2^x$ and the line $l(x) = x+1$. Conjecture G is says that the graph of the expontial function f is "strictly higher" than the graph of the line function l for all $x > 0$. Note $f(0) = 1 = l(0)$ and $f(1) = 2 = l(1)$ so both functions go through the points $(0,1)$ and $(1,2)$. Thus when $x = 0$ and $x = 1$, get $f(x) = l(x)$. The graph shows that when $x \in [0,1]$, get $f(x) \not> l(x)$. Keeping in mind that Conjecture G has $\left(\forall x \in \mathbb{R}^{>0}\right)$, we see that any $x \in (0,1]$ would provide a counterexample. To make the arithmetic easy, let's take $x_0 = 1$.

Counterexample. Conjecture G is false, as shown by the counterexample $x_0 = 1$ since

$$2^{x_0} = 2^1 = 2 \not> 2 = 1 + 1 = x_0 + 1.$$

H. **Conjecture H**. For every positive real number x, there is a positive real number y less than x with the property that for all positive real numbers z, we have $yz \geq z$.

H1. Symbolically write Conjecture H. As universes, use $\mathbb{R}^{>0}$

$$\boxed{\left(\forall x \in \mathbb{R}^{>0}\right)\ \left(\exists y \in \mathbb{R}^{>0}\right)\ \left[\ y < x\ \wedge\ \left(\forall z \in \mathbb{R}^{>0}\right)\ [yz \geq z]\ \right]}$$

H2. Symbolically write an useful negation of Conjecture H, without using $\sim$ nor $\neg$ nor $\wedge$ nor $\vee$.

$$\boxed{\left(\exists x \in \mathbb{R}^{>0}\right)\ \left(\forall y \in \mathbb{R}^{>0}\right)\ \left[\ y < x\ \Rightarrow\ \left(\exists z \in \mathbb{R}^{>0}\right)\ [yz < z]\ \right]}$$

Rmk: used $[\ \sim (P \wedge Q)\ ] \equiv [\ P \Rightarrow (\sim Q)\ ]$ rather than DeMorgan's $[\sim (P \wedge Q)\ ] \equiv [(\sim P) \vee (\sim Q)]$. A solution using DeMorgan's would also be correct.

H3. Is Conjecture H true or false? (circle one) TRUE $\boxed{\text{FALSE}}$.

H4. Is the negation of Conjecture H true or false? (circle one) $\boxed{\text{TRUE}}$ FALSE .

H5. Provide a proof or a counterexample to Conjecture H.

Thinking Land. In H1 look at the part $\left(\forall z \in \mathbb{R}^{>0}\right)[yz \geq z]$. Since $z > 0$, we can divide by z to see $[yz \geq z] \Leftrightarrow [y \geq 1] \Leftrightarrow [1 \leq y]$. Thus statement in H1 is logically equivalent to

$$\left(\forall x \in \mathbb{R}^{>0}\right)\ \left(\exists y \in \mathbb{R}^{>0}\right)\ \left[\ y < x\ \wedge\ \left(\forall z \in \mathbb{R}^{>0}\right)\ [1 \leq y]\ \right]$$

So if Conjecture H were true, we would have, for each $x > 0$ there exists $y > 0$ such that $1 \leq y < x$, which would imply that for any $x > 0$ we take, $1 < x$. Wow, we just found the counterexample. If we pick $x = 1$ then $x > 0$ but $1 \not< x$.

Counterexample. Conjecture H is false, as shown by the counterexample $x_0 = 1$. Assume that there exists $y_0 \in \mathbb{R}^{>0}$ with $y_0 < x_0$ and, for each $z > 0$, we have $y_0 z \geq z$. Since $y_0 < x_0$

$$y_0 < 1. \tag{1}$$

Since for each $z > 0$, we have $z \leq y_0 z$, taking $z = 1$ gives

$$1 \leq y_0 \tag{2}$$

Combining (1) and (2) gives $1 \leq y_0 < 1$, which implies $1 < 1$. But $1 \not< 1$.

Thus $x_0 = 1$ is a counterexample to Conjecture H.

An agruement that Conjecture H is false. First we argue the negation to Conjecture H is true. A negation to Conjecture H is, symbolically written,

$$\left(\exists x \in \mathbb{R}^{>0}\right)\ \left(\forall y \in \mathbb{R}^{>0}\right)\ \left[\ y < x\ \Rightarrow\ \left(\exists z \in \mathbb{R}^{>0}\right)\ [yz < z]\ \right].$$

Take $x_0 = 1$. Clearly $x_0 \in \mathbb{R}^{>0}$. Let $y \in \mathbb{R}^{>0}$ and $y < x_0$. Thus $y < 1$. So for each $z \in \mathbb{R}^{>0}$ we have that $yz < z$. We have just shown that the negation to Conjecture H is true.

Thus, since the negation to Conjecture H is true, Conjecture H is false.

I. **Conjecture I**. For every positive real number x, there is a positive real number y with the property that if $y < x$, then for all positive real numbers z, it holds that $yz \geq z$.

I1. Symbolically write Conjecture I.

$$\boxed{\left(\forall x \in \mathbb{R}^{>0}\right)\ \left(\exists y \in \mathbb{R}^{>0}\right)\ \left[\ y < x\ \Rightarrow\ \left(\forall z \in \mathbb{R}^{>0}\right)\ [yz \geq z]\ \right]}$$

I2. Is Conjecture I true or false? (circle one) $\boxed{\text{TRUE}}$ FALSE .

I3. Provide a proof or a counterexample to Conjecture I.

Thinking Land. Fix $x \in \mathbb{R}^{>0}$. Choose $y = x$. So $y \in \mathbb{R}^{>0}$. Then the conditional statement

$$\left[\ y < x\ \Rightarrow\ \left(\forall z \in \mathbb{R}^{>0}\right)\ [yz \geq z]\ \right]$$

is true since the hypothesis $y < x$ is false.

Proof of Conjecture I. Towards showing Conjecture I is true, fix $x \in \mathbb{R}^{>0}$. We will show that there exists $y \in \mathbb{R}^{>0}$ such that

$$\text{if } y < x \text{ then for each } z \in \mathbb{R}^{>0} \text{ we have } yz \geq z. \tag{1}$$

Let $y = x$. Then the hypothesis $y < x$ is the conditional statement in (1) is false. Thus the conditional statement in (1) is true.

We have just proved that Conjecture I is true. $\square$

Alternative Thinking Land. Fix $x \in \mathbb{R}^{>0}$. Choose $y = 1$. Then if $z \in \mathbb{R}^{>0}$ then $yz = z$ and so $yz \leq z$.