Strong Induction (also called complete induction, our book calls this 2nd PMI) §4.2 p194

Fix $n_0 \in \mathbb{Z}$.
If

BASE STEP: $P(n_0)$ is true

INDUCTIVE STEP: for each $n \in \mathbb{Z}^{\geq n_0}$: $\underbrace{[\ P(j) \text{ is true for } j \in \{n_0, 1+n_0, \ldots, n\}\ ]}_{\text{inductive hypothesis}} \Rightarrow \underbrace{[\ P(n+1) \text{ is true}\ ]}_{\text{inductive conclusion}}$

then $P(n)$ is true for each $n \in \mathbb{Z}^{\geq n_0}$.

Ex. **Theorem**. Let $\{a_n\}_{n=0}^{\infty}$ be the recursively defined sequence of integers

$$a_0 = 2 \quad , \quad a_1 = 4 \quad , \quad a_2 = 6$$

and

$$a_n = 5a_{n-3} \quad \text{when} \quad n \in \mathbb{N} \text{ and } n \geq 3. \tag{RD}$$

Then a_n is even for each $n \in \mathbb{Z}^{\geq 0} \overset{\text{i.e.}}{=} \{0, 1, 2, 3, 4, \ldots\}$. RD = Recursive Def. ↑

►. Symbolically:

Thinking Land

Let's make a chart to help us understand better what is going on.

n	a_n
0	2 (given)
1	4 (given)
2	6 (given)
now the recursive definition *kicks in*	
3	
4	
5	
6	
7	
8	
Do we see a pattern?	

∘. For the Base Step, which n's do we need to check? ________________.

∘. Since in the Base Step we verified the Thm. holds up to (and including) $n =$ ___,

where should we start the Induction Step? At $n =$ ___.

So the first line in your induction step should look something line:

For the inductive step, fix $n \in \mathbb{N}$ such that $n \geq$ ___. Assume the inductive hypothesis, which is

(IH)

We will show the inductive conclusion, which is

(IC)

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Fix $n_0 \in \mathbb{Z}$.
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BASE STEP: $P(n_0)$ is true
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then $P(n)$ is true for each $n \in \mathbb{Z}^{\geq n_0}$.

Theorem. Each natural number n has a factorization as

$$n = 2^k m$$

for some k is some nonnegative integer and some odd natural number m.

►. Written symbolically:

Thinking Land

Let's make a chart to help us understand better what is going on.

n	$n = 2^k m$ where $k \in \{0, 1, 2, 3, 4, 5, \ldots\}$ and $m \in \{1, 3, 5, 7, 9, 11, \ldots\}$
1	$1 =$
2	$2 =$
3	$3 =$
4	$4 =$
5	$5 =$
6	$6 =$
7	$7 =$
So if n is _____, then $n =$ _____ so $k :=$ __ $\in \mathbb{Z}^{\geq 0}$ and $m :=$ __ with m an odd natural number.	
8	$8 =$
10	$10 =$
12	$12 =$

◦. For the Base Step, which n's do we need to check? ______________________________.

◦. Since in the Base Step we verified the Thm. holds up to (and including) $n =$ ____,

where should we start the Induction Step? At $n =$ ____.

So the first line in your induction step should look something line:

For the inductive step, fix $n \in \mathbb{N}$ such that $n \geq$ ____. Assume the inductive hypothesis, which is

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(IC)

To show the (IC), we will need to consider ⟨the only possible⟩ two cases for n: ________ and ________.