Let $\mathbb{N} = \{1, 2, 3, \ldots \}$ be the natural numbers.
Let $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots \}$ be the integers.
Let $P(n)$ be a statement (that is either true or false) about $n$.
Sometimes we denote $P(n)$ by $P_n$.

§ 2.4: PMI (basic form)

If

- **BASE STEP:** $P(1)$ is true
- **INDUCTIVE STEP:** for each $n \in \mathbb{N}$: \[ P(n) \text{ is true } \implies P(n + 1) \text{ is true } \]

then $P(n)$ is true for each $n \in \mathbb{N}$.

§ 2.4: PMI (generalized form) (also known as: doesn’t matter where you start form)

Fix $n_0 \in \mathbb{Z}$.
If

- **BASE STEP:** $P(n_0)$ is true
- **INDUCTIVE STEP:** for each $n \in \mathbb{Z}$ with $n \geq n_0$: \[ P(n) \text{ is true } \implies P(n + 1) \text{ is true } \]

then $P(n)$ is true for each $n \in \mathbb{Z}$ such that $n \geq n_0$.

§ 2.5: PMI (strong form) (our book calls this PCI = Principle of Complete Induction)

Fix $n_0 \in \mathbb{Z}$.
If

- **BASE STEP:** $P(n_0)$ is true
- **INDUCTIVE STEP:** for each $n \in \mathbb{Z}$ with $n \geq n_0$:
  \[ P(j) \text{ is true for } j = n_0, 1 + n_0, \ldots, n \implies P(n + 1) \text{ is true } \]

then $P(n)$ is true for each $n \in \mathbb{Z}$ such that $n \geq n_0$. 