Thm. **Theorem 17**. For each $n \in \mathbb{N}$

$$\sum_{i=1}^{n} i\,(i!) \;=\; (n+1)! - 1\ .$$

►. Using math induction, prove Theorem 17.

○. Hints.
Recall $n! = 1 \cdot 2 \cdots (n-1) \cdot n = n \cdot (n-1) \cdot (n-2) \cdots 2 \cdot 1$.
So $(n!)\,(n+1) = (1 \cdot 2 \cdots n)\,(n+1) = (n+1)!$.
Don't forget needed parentheses: $(n!)(n+1) \neq (n!)n + 1$.

. . . .

$\boxed{\text{BS}}$. Let $n = 1$.

$$\sum_{i=1}^{n} i\,(i!) \;=\; \sum_{i=1}^{1} i\,(i!) \;=\; 1(1!) \;=\; 1(1) \;=\; 1$$

and

$$(n+1)! - 1 \;=\; (1+1)! - 1 \;=\; (2)! - 1 \;=\; 2 - 1 \;=\; 1$$

$\boxed{\text{IS}}$. Fix $n \in \mathbb{N}$. Let

$$\sum_{i=1}^{n} i\,(i!) \;=\; (n+1)! - 1\ . \tag{IH}$$

WTS

$$\sum_{i=1}^{n+1} i\,(i!) \;=\; (\ (n+1) + 1\ )! - 1\ .$$

i.e.,

$$\sum_{i=1}^{n+1} i\,(i!) \;=\; (n+2)! - 1\ . \tag{IC}$$

Well

$$\sum_{i=1}^{n+1} i\,(i!) \;=\; \left[\sum_{i=1}^{n} i\,(i!)\right] + (n+1)\,(n+1)!$$

and by (IH)

$$=\ [\ (n+1)! - 1\ ] + (n+1)\,(n+1)!$$

now just algebra

$$=\ (n+1)! + (n+1)\,(n+1)! \quad -1$$

$$=\ (n+1)!\,[\ 1 + (n+1)\ ] \quad -1$$

$$=\ (n+1)!\,[\ n+2\ ] \quad -1$$

$$=\ (n+2)! \quad -1.$$