INSTRUCTIONS:

(1) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.

(2) When applicable put your answer on/in the line/box provided. Show your work UNDER the provided line/box.

(3) For problems that request a proof, write a neat FORMAL proof and use only logic and the definitions of the concepts involved (i.e., do not quote a problem from the book). You may include your skeleton, if you so wish.

(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.

(5) You may not use a calculator, books, personal notes.

(6) This exam covers (from A Transition to Advanced Mathematics by Smith, Eggen, and St. Andre: 6th ed.): Sections 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 2.1, 2.2.

Numbers:

- real numbers = \( \mathbb{R} \)
- positive real numbers = \( \mathbb{R}^+ = \{ x \in \mathbb{R}: x > 0 \} \)
- natural numbers = \( \mathbb{N} = \{ 1, 2, 3, 4, \ldots \} \)
- integers = \( \mathbb{Z} = \{ 0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots \} \)
- rational numbers = \( \mathbb{Q} = \left\{ \frac{p}{q} \in \mathbb{R}: p, q \in \mathbb{Z} \text{ and } q \neq 0 \right\} \)

Throughout this exam:

(1) \( P, Q, \) and \( R \) be propositions.

(2) \( P(x_1, \ldots, x_n), Q(x_1, \ldots, x_n), \) and \( R(x_1, \ldots, x_n) \) are open sentences with variables \( x_1, \ldots, x_n \).
1. Make true statements by filling in the blanks with the appropriate number between 1 and 11 from below. Use each number once, and only once.

(1) \( (P \implies Q) \land (P \implies R) \)
(2) \( (P \implies Q) \land (Q \implies P) \)
(3) \( \sim Q \implies \sim P \)
(4) \( (P \land \sim Q) \implies R \)
(5) \( (P \land \sim Q) \)
(6) \( (\sim P) \land (\sim Q) \)
(7) \( (P \implies R) \land (Q \implies R) \)
(8) \( P \implies \sim Q \)
(9) \( (\forall x)[\sim P(x)] \)
(10) \( (\exists x)[\sim P(x)] \)
(11) \( (\exists x)[P(x) \land (\forall y)(P(y) \implies x = y)] \)

\( P \implies Q \) is equivalent to ____________
\( P \iff Q \) is equivalent to ____________
\( \sim (P \implies Q) \) is equivalent to ____________
\( \sim (P \land Q) \) is equivalent to ____________
\( \sim (P \lor Q) \) is equivalent to ____________
\( P \implies (Q \land R) \) is equivalent to ____________
\( P \implies (Q \lor R) \) is equivalent to ____________
\( (P \lor Q) \implies R \) is equivalent to ____________
\( \sim [(\forall x)P(x)] \) is equivalent to ____________
\( \sim [(\exists x)P(x)] \) is equivalent to ____________
\( (\exists ! x)P(x) \) is equivalent to ____________
Circle T if and only if the statement is TRUE. Circle F if and only if the statement if FALSE.

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3. Show that

\[ \neg (P \implies Q) \text{ is equivalent to } (P \land \neg Q) \]

by making a truth table and then writing a sentence or so about how you are reading your truth table to come up with your answer.
4. Do THREE problems from problems: 4a, 4b, 4c, and 4d. Fill-in the blanks on this sheet of paper but include your proof parts on a separate sheet of paper (with your name on each sheet please).

Circle THREE: I am doing problems 4a , 4b , 4c , 4d

4a. Let \( x \in \mathbb{Z} \). If 8 does not divide \( x^2 - 1 \), then \( x \) is even.

- Let \( a, b, c \in \mathbb{Z} \). By definition:

  \( a \) divides \( b \), denoted by \( a | b \), if and only if __________________________.

  \( c \) is even if and only if __________________________.

  \( c \) is odd if and only if __________________________.

- Symbolically write, using quantifiers, statement 4a.

- Prove statement 4a.

4b. There exists an odd integer \( M \) such that for each real number \( r \) larger than \( M \), \( \frac{1}{25r} < 0.01 \).

- By definition, an integer \( M \) is odd if and only if __________________________.

- Symbolically write, using quantifiers, statement 4b.

- Prove statement 4b.
4c. For each rational number $x$ and each irrational number $y$, there is a unique irrational number $z$ such that $y + z = x$.

- Let $a$ and $b$ be real numbers. By definition:

  $a$ is rational if and only if __________

  $b$ is irrational if and only if __________.

- Symbolically write, using quantifiers, statement 4c.

- Prove statement 4c. You must carefully prove, using only the definitions above (and not using another homework problem), that your $z$ is indeed irrational.

4d. Let $A, B, C$ be subsets of the universe $U$. Show that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

- By definition:

  $x \in A \cup B$ if and only if __________

  $x \in B \cap C$ if and only if __________

- Let $P, Q, R$ be propositions. The distributive rule says that

  $[P \land (Q \lor R)]$ is logically equivalent to __________

  $[P \lor (Q \land R)]$ is logically equivalent to __________

- Prove statement 4d. You can use the logical equivalents for propositions that you just wrote above.