NAME:____

Prof. Girardi	11.30.06	Exam 2	Math 300

MARK BOX		
PROBLEM	POINTS	
1 (parts 1 – 32)	32	
2abc	12	
3abc	12	
4abc	12	
5	10	
6	11	
7	11	
TOTAL	100	

INSTRUCTIONS:

(1) The MARK BOX indicates the problems along with their points.
Check that your copy of the exam has all of the problems.

(2) When applicable put your answer on/in the line/box provided.
Show your work **UNDER** the provided line/box.
If no such line/box is provided, then box your answer.

(3) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.

(4) You may **not** use a calculator, books, personal notes.

(5) This exam covers (from *A Transition to Advanced Mathematics* by Smith, Eggen, and St. Andre: 6^{th} ed.): Sections 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7 .

Numbers:

- real numbers $= \mathbb{R}$
- natural numbers $= \mathbb{N} = \{1, 2, 3, 4, \ldots\}$
- integers $= \mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots\}$
- rational numbers $= \mathbb{Q} = \left\{ \frac{p}{q} \in \mathbb{R} \colon \ p, q \in \mathbb{Z} \text{ and } \ q \neq 0 \right\}$

Throughout this exam:

(1) P, Q, and R are propositions.

(2) $P(x_1, x_2, \ldots, x_n)$, $Q(x_1, x_2, \ldots, x_n)$, and $R(x_1, x_2, \ldots, x_n)$ are open sentences with variable $x_1, x_2, \ldots, x_n$.

1. This problem has numbers 1 to 32.

- On numbers 1 – 19, fill in the blanks with A – S from the next page to make a tautology.

  Use each letter once, and only once.

⋆ 1.	$[\ P \wedge (P \Rightarrow Q)\ ]$	$\Longrightarrow$	________	(modus ponens)
⋆ 2.	$[\ (P \Rightarrow Q) \wedge (Q \implies R\ ]$	$\Longrightarrow$	________	(transitivity)
⋆ 3.	$[\ P \Leftrightarrow Q\ ]$	$\Longleftrightarrow$	________	(biconditional)
⋆ 4.	$[\ \sim P \implies (Q \wedge \sim Q)\ ]$	$\Longleftrightarrow$	________	(contradiction)
⋆ 5.	$[\ P \vee (Q \vee R)\ ]$	$\Longleftrightarrow$	________	(associativity)
⋆ 6.	$[\ P \wedge (Q \wedge R)\ ]$	$\Longleftrightarrow$	________	(associativity)
⋆ 7.	$[\ P \wedge (Q \vee R)\ ]$	$\Longleftrightarrow$	________	(distributive rule)
⋆ 8.	$[\ P \vee (Q \wedge R)\ ]$	$\Longleftrightarrow$	________	(distributive rule)
⋆ 9.	$[\ \sim (P \vee Q)\ ]$	$\Longleftrightarrow$	________	(DeMorgan's Law)
⋆ 10.	$[\ \sim (P \wedge Q)\ ]$	$\Longleftrightarrow$	________	(DeMorgan's Law)
⋆ 11.	$[\ \sim (P \wedge Q)\ ]$	$\Longleftrightarrow$	________	(denial of and)
⋆ 12.	$[\ \sim (P \implies Q)\ ]$	$\Longleftrightarrow$	________	(denial of implication)
⋆ 13.	$[\ P \implies (Q \wedge R)\ ]$	$\Longleftrightarrow$	________	
⋆ 14.	$[\ P \implies (Q \vee R)\ ]$	$\Longleftrightarrow$	________	
⋆ 15.	$[\ (P \wedge Q) \implies R\ ]$	$\Longleftrightarrow$	________	
⋆ 16.	$[\ (P \vee Q) \implies R\ ]$	$\Longleftrightarrow$	________	
⋆ 17.	$\sim [(\forall x)P(x)]$	$\Longleftrightarrow$	________	
⋆ 18.	$\sim [(\exists x)P(x)]$	$\Longleftrightarrow$	________	
⋆ 19.	$(\exists! x)P(x)$	$\Longleftrightarrow$	________	

- On 20 and 21, fill in the blank with <u>is equivalent to</u> or <u>is not equivalent to</u>

  ⋆ 20. $P \Rightarrow Q$ ________________________________ $(\sim Q) \Rightarrow (\sim P)$.

  ⋆ 21. $P \Rightarrow Q$ ________________________________ $Q \Rightarrow P$

- On 22 and 23, fill in the blank with <u>converse</u> or <u>contrapositive</u>.

  ⋆ 22. The ________________________________ of $P \Rightarrow Q$ is: $(\sim Q) \Rightarrow (\sim P)$.

  ⋆ 23. The ________________________________ of $P \Rightarrow Q$ is: $Q \Rightarrow P$.

- On 24 and 32, circle T if the statement is a tautology and F if the statement is not a tautology.

⋆ 24. $(\exists x)P(x)$	$\Rightarrow$	$(\exists! x)P(x)$	T	F
⋆ 25. $(\exists! x)P(x)$	$\Rightarrow$	$(\exists x)P(x)$	T	F
⋆ 26. $(\forall x)(\forall y)P(x,y)$	$\Leftrightarrow$	$(\forall y)(\forall x)P(x,y)$	T	F
⋆ 27. $(\exists x)(\exists y)P(x,y)$	$\Leftrightarrow$	$(\exists y)(\exists x)P(x,y)$	T	F
⋆ 28. $(\exists x)(\forall y)P(x,y)$	$\Leftrightarrow$	$(\forall y)(\exists x)P(x,y)$	T	F
⋆ 29. $(\exists x)P(x)$	$\Leftrightarrow$	$(\forall x)P(x)$	T	F
⋆ 30. $(\forall x)[P(x) \implies Q(x)]$	$\Leftrightarrow$	$[(\forall x)P(x) \implies (\forall x)Q(x)]$	T	F
⋆ 31. $(\forall x)[P(x) \wedge Q(x)]$	$\Leftrightarrow$	$[(\forall x)P(x) \wedge (\forall x)Q(x)]$	T	F
⋆ 32. $[(\forall x)P(x) \vee (\forall x)Q(x)]$	$\Leftrightarrow$	$(\forall x)[P(x) \vee Q(x)]$	T	F

For problem 1 on the previous page, parts 1 - 19. Here is A – S.

- ⋆ A. $[\ P\ ]$
- ⋆ B. $[\ Q\ ]$
- ⋆ C. $[\ (\sim P) \vee (\sim Q)\ ]$
- ⋆ D. $[\ (\sim P) \wedge (\sim Q)\ ]$
- ⋆ E. $[\ P \wedge \sim Q\ ]$
- ⋆ F. $[\ P \implies R\ ]$
- ⋆ G. $[\ P \implies \sim Q\ ]$
- ⋆ H. $[\ (P \wedge Q) \vee (P \wedge R)\ ]$
- ⋆ I. $[\ (P \vee Q) \wedge (P \vee R)\ ]$
- ⋆ J. $[\ (P \wedge Q) \wedge R\ ]$
- ⋆ K. $[\ (P \vee Q) \vee R\ ]$
- ⋆ L. $[\ (P \wedge \sim Q) \implies R\ ]$
- ⋆ M. $[\ (P \wedge \sim R) \implies \sim Q\ ]$
- ⋆ N. $[\ (P \implies Q) \wedge (P \implies R)\ ]$
- ⋆ O. $[\ (P \implies R) \wedge (Q \implies R)\ ]$
- ⋆ P. $[\ (P \implies Q) \wedge (Q \implies P)\ ]$
- ⋆ Q. $(\exists x)P(x) \quad \wedge \quad (\forall y)(\forall z)(P(y) \wedge P(z) \Rightarrow y = z)$
- ⋆ R. $(\forall x)[\sim P(x)]$
- ⋆ S. $(\exists x)[\sim P(x)]$

2. Write symbolically, using quantifiers.

2a. For all integers n, $5n^2 + 3n + 1$ is odd.

2b. For all odd integers n, $2n^2 + 3n + 4$ is odd.

2c. If x is a rational number and y is an irrational number, then $x + y$ is irrational.

3. Write symbolically, using quantifiers.

3a. For every odd integer m, if m has the form $4k + 1$ for some integer k, then $m + 2$ has the form $4j - 1$ for some integer j.

3b. For every odd integer m, $m^2 = 8k + 1$ for some integer k.

3c. Let a, b, and c be integers. If a divides $b - 1$ and a divides $c - 1$, then a divides $bc - 1$.

4. Write symbolically, using quantifiers.
(Recall, $\mathbb{R}^+ = \{r \in \mathbb{R} \colon r > 0\}$ = the set of positive real numbers.)

4a. For every positive real number x, there is a positive real number y with the property that if $y < x$, then for all positive real numbers z, the inequality $yz \geq z$ is true.

4b. For every natural number a there is a natural number b such that $2a < b$.

4c. If x and y are rational numbers with $x < y$, there exists a rational number z so that $x < z < y$.

For problems 5 – 7, use the scratch paper provided and put your name on each page. A skeleton is optional (for partial credit basically). Prof. G would appreciate space between lines to make comments.

5. Prove ONE of the THREE statements in problem 2.

Circle one: I am proving number: 2a 2b 2c .

6. Prove ONE of the THREE statements in problem 3.

Circle one: I am proving number: 3a 3b 3c .

7. Prove ONE of the THREE statements in problem 4.

Circle one: I am proving number: 4a 4b 4c .