INSTRUCTIONS:

(1) Do 3 of the 4 problems. I am doing problem numbers: ____________________________.

(2) Use your own paper or the scratch paper I provided:
   - write on only one side of the page
   - begin each (numbered) problem on a new page
   - put your name on each page

(3) For problems that request a proof, write a neat formal proof and use only logic and the definitions of the concepts involved (i.e., do not quote a problem from the book). You may include your skeleton, if you so wish, but points will be given for your formal proof only.

(4) The MARK BOX indicates the problems along with their points.

   Check that your copy of the exam has all of the problems.

(5) During this exam, do not leave your seat. If you have a question, raise your hand.

   When you finish: turn your exam over, put your pencil down, and raise your hand.

(6) This closed book/notes exam covers (from A Transition to Advanced Mathematics by Smith, Eggen, and St. Andre: 5th ed.): Chapter 1.

Problem Inspiration:

1. similar to look-at problem § 1.2 # 7e
2. hand-in problem § 1.5 # 3d
3. look-at problem § 1.6 # 2b
4. in-class problem § 1.7 # 3a

Numbers:

- real numbers = \( \mathbb{R} \)
- natural numbers = \( \mathbb{N} = \{1, 2, 3, 4, \ldots\} \)
- integers = \( \mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots\} \)
- rational numbers = \( \mathbb{Q} = \left\{ \frac{p}{q} \in \mathbb{R} : p, q \in \mathbb{Z} \text{ and } q \neq 0 \right\} \)
1. Make a truth table for the propositional form:

\[
[ \sim (P \Rightarrow Q) ] \Leftrightarrow [ P \land \sim Q ] .
\]

Is this propositional form a tautology, a contradiction, or neither? Explain your answer in a complete sentence.

2. Let \( x \) and \( y \) be integers. Show that if \( xy \) is even, then \( x \) is even or \( y \) is even.

2a. First symbolically write, using quantifiers, the statement to be proved.

2b. Give a formal proof of the statement to be proved.

Recall: By definition, for an integer \( z \):

- \( z \) is even if \( z = 2k \) for some \( k \in \mathbb{Z} \)
- \( z \) is odd if \( z = 2j + 1 \) for some \( j \in \mathbb{Z} \).

Hint: contrapositive

3. Let \( a \), \( b \), and \( c \) be integers. Show that if \( a \) divides \( b - 1 \) and \( a \) divides \( c - 1 \) then \( a \) divides \( bc - 1 \).

3a. First symbolically write, using quantifiers, the statement to be proved.

3b. Give a formal proof of the statement to be proved.

Recall: By definition, for integers \( x \) and \( y \) where \( x \neq 0 \), \( x \) divides \( y \) if \( kx = y \) for some \( k \in \mathbb{Z} \).

Hint: \( bc - 1 = (b - 1)(c - 1) + (b - 1) + (c - 1) \)

4. Prove that if \( x \) is rational and \( y \) is irrational then \( x + y \) is irrational.

4a. First symbolically write, using quantifiers, the statement to be proved.

4b. Next symbolically write, using quantifiers, a (useful) denial of your symbolic statement in 4a.

4c. Give a formal proof of the statement to be proved.

Recall:

- the definition of rational number is given on page 1 of this test
- \( y \) is irrational if and only if \( y \in \mathbb{R} \setminus \mathbb{Q} \)

Hint: contradiction. Assume that 4a. does not hold. Thus 4b. holds. Now find a contradiction.