► If needed, look at the LaTeX for this Exercise to help out with the LaTex for other Math Induction exercises.
► Most of the work done in an induction proof is usually in proving the inductive step. This was certainly the case in Proposition 4.2 (pg. 175). However, the basis step is an essential part of the proof. As this Exercise illustrates, an induction proof is incomplete without the Base Step.

Exercise. A variant of Exercise 4.1.19. **The Importance of the Basis Step**. §4.1 p186

Let $P(n)$ be ⟨the open sentence in the variable $n \in \mathbb{N}$⟩

$$\sum_{j=1}^{n} j = \frac{n^2+n+1}{2}. \tag{$P(n)$}$$

i. Let $n \in \mathbb{N}$. Complete the following proof that if $P(n)$ is true then $P(n+1)$ is true (by cutting out the "for you ... " parts and replacing them with the needed algebra). Note the proof and exercise are continued next page.

Proof. Let $P(n)$ be the open sentence in the variable $n \in \mathbb{N}$

$$\sum_{j=1}^{n} j = \frac{n^2+n+1}{2}.$$

Fix $n \in \mathbb{N}$. Assume $P(n)$ is true ⟨think of as the inductive hypothesis⟩. Thus we are assuming

$$\sum_{j=1}^{n} j = \frac{n^2+n+1}{2}. \tag{IH}$$

We shall show that $P(n+1)$ is true ⟨think of as the inductive conclustion⟩. That is, we shall show that

$$\sum_{j=1}^{n+1} j = \frac{(n+1)^2+(n+1)+1}{2}. \tag{IC}$$

Thus we have

$$\sum_{j=1}^{n+1} j = \left[\sum_{j=1}^{n} j\right] + (n+1)$$

and by (IH) we get (note how this is LaTexed with intertext)

$$= \left[\frac{n^2+n+1}{2}\right] + (n+1)$$

and now by algebra

$$= \text{for you ... get a common demoninator}$$

$$= \text{for you ... now keep doing simple algebra ...}$$

$$= \text{for you ... use as many lines as you need ...}$$

$$= \text{for you ... until you get the Right Hand Side of (IC) ... .}$$

We have just show that (IC) holds.

Thus, for each $n \in \mathbb{N}$, if $P(n)$ is true then $P(n+1)$ is true. ■

ii. Is $P(1)$ true? Is $P(2)$ true? Using Progress Check 4.3 (see pages 177 and 510), for what $n \in \mathbb{N}$ is $P(n)$ true? Explain how this shows that the basis step is an essential part of a proof by induction.