

Recall that the distance between the two points $(x_1, y_1) \in \mathbb{R}^2$ and $(x_2, y_2) \in \mathbb{R}^2$ in the plane is commonly denoted by $d((x_1, y_1), (x_2, y_2))$ and has the formula

$$d((x_1, y_1), (x_2, y_2)) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

If you did not immediately remember this, just look at the pictures at this precalculus website.

Hint. The Latex Lesson on aligning formula (Lesson Number 2) will be helpful.

Theorem 1. If $x < -4$ and $y > 2$, then the distance between (x, y) and $(1, -2)$ is strictly larger than 6.

Exercise. A variant of Exercise 3.1.101.

§3.1

Finish the below proof of Theorem 1. Use simple high school algebra. You may not use concepts from calculus. Drawing a picture (with a triangle as in the precalculus website) might help.

Proof. Fix $(x, y) \in \mathbb{R}^2$ such that

$$x < -4$$

and

$$y > 2.$$

We shall show that the distance between (x, y) and $(1, -2)$ is strictly larger than 6.

Since

$$x < -4$$

we have that

$$x - 1 < -5$$

and so

$$(x - 1)^2 \text{ ? ? ? fill in through here} \tag{1}$$

Students ... keep going ...

