▶ If needed, look at the LaTeX for Exercise 1 to help out with the LaTex for Exercise 2.

Exercises

Exercise 1. A variant of Exercise 4.1.19. **The Importance of the Basis Step**. p186

Most of the work done in constructing a proof by induction is usually in proving the inductive step. This was certainly the case in Proposition 4.2. However, the basis step is an essential part of the proof. Without it, the proof is incomplete. To see this, let $P(n)$ be ⟨the open sentence in the variable $n \in \mathbb{N}$⟩

$$\sum_{j=1}^{n} j = \frac{n^2+n+1}{2}. \qquad (P(n))$$

ER 1i. Let $n \in \mathbb{N}$. Complete the following proof that if $P(n)$ is true then $P(n+1)$ is true.

Proof. Let $P(n)$ be the open sentence in the variable $n \in \mathbb{N}$

$$\sum_{j=1}^{n} j = \frac{n^2+n+1}{2}.$$

Fix $n \in \mathbb{N}$. Assume $P(n)$ is true ⟨think of as the inductive hypothesis⟩. Thus we are assuming

$$\sum_{j=1}^{n} j = \frac{n^2+n+1}{2}. \qquad \text{(IH)}$$

We shall show that $P(n+1)$ is true ⟨think of as the inductive conclustion⟩. That is, we shall show that

$$\sum_{j=1}^{n+1} j = \frac{(n+1)^2+(n+1)+1}{2}. \qquad \text{(IC)}$$

Thus we have

$$\sum_{j=1}^{n+1} j = \left[\sum_{j=1}^{n} j\right] + (n+1)$$

and by (IH) we get

$$= \left[\frac{n^2+n+1}{2}\right] + (n+1)$$

and now by algebra

$$= \text{for you ... get a common demoninator}$$

$$= \text{for you ... now keep doing simple algebra ...}$$

$$= \text{for you ... use as many lines as you need ...}$$

$$= \text{for you ... until you get the Right Hand Side of (IC) ... .}$$

We have just show that (IC) holds.

Thus, for each $n \in \mathbb{N}$, if $P(n)$ is true then $P(n+1)$ is true. . ■

ER 1ii. . Is $P(1)$ true? Is $P(2)$ true? Using Progress Check 4.3 (see pages 177 and 510), for what $n \in \mathbb{N}$ is $P(n)$ true? Explain how this shows that the basis step is an essential part of a proof by induction.

Exercise 2. A variant of Exercise 4.1.3c. p180

Using math induction, prove that for each $n \in \mathbb{N}$

$$\sum_{j=1}^{n} j^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

Hint. On the course handout page, look through the Student Solutions posted to the Ch. 4 Induction Group Work (from a previous semester) as to learn from their mistakes.