▶ Note due at 11pm, not 11:59pm.

▶ Recall, when asked to *symbolically write* a statement, do so **using quanifiers**.

▶ Unless otherwise stated, in a proof you may use any previous result (or Exercise) provided you reference the result.

▶ If you need, look at the LaTeX here to remind yourself how to Latex:

$n|a$ (i.e., n divides a), $n \nmid a$ (i.e., n does not divide a), and $a \equiv\ b \pmod{n}$ (i.e., a is congruent to b modulo n).

Exercises

Exercise 1. A variant of Exercise 3.3.6a. p126

Conjecture 1. For each positive real number x, if x is irrational, then x^2 is irrational.

ER 1i. Sybolically write Conjecture 1.

ER 1ii. Determine if Conjecture 1 is true or false. If Conjecture 1 is true, then write a formal proof of Conjecture 1. If Conjecture 1 is false, then provide a counterexample that shows (and clearly explains) why Conjecture 1 if false.

Exercise 2. A variant of Exercise 3.3.6c. p127

Conjecture 2. For every pair of real numbers x and y, if $x + y$ is irrational, then x is irrational and y is irrational.

ER 2i. Sybolically write Conjecture 2.

ER 2ii. Determine if Conjecture 2 is true or false. If Conjecture 2 is true, then write a formal proof of Conjecture 2. If Conjecture 2 is false, then provide a counterexample that shows (and clearly explains) why Conjecture 2 if false.

Exercise 3. A variant of Exercise 3.3.8a. p127

Theorem 3. If x is a real number, then $\left(x + \sqrt{2}\right)$ is irrational or $\left(-x + \sqrt{2}\right)$ is irrational.

ER 3i. Sybolically write Theorem 3. Hint: Use $\mathbb{R}$ as the universe.

ER 3ii. Prove Theorem 3. Hint: You may use Theorem 3.20.

Exercise 4. A variant of Exercise 3.4.5a. p138

Theorem 4. For all integers a, b, and d with $d \neq 0$, if d divides a or d divides b, then d divides the product ab.

ER 4i. Sybolically write Theorem 4.

ER 4ii. Prove Theorem 4. Hint: Notice that the hypothesis is a disjunction. So use two cases.