- ▶ Note due at 11pm, not 11:59pm.
- ▶ Recall, when asked to *symbolically write* a statement, do so **using quanifiers**.
- ▶ Unless otherwise stated, in a proof you may use any previous result (or Exercise) provided you reference the result.
- ▶ If you need, look at the LaTeX here to remind yourself how to Latex:

  $n|a$ (i.e., n divides a), $n \nmid a$ (i.e., n does not divide a), and $a \equiv\ b \pmod{n}$ (i.e., a is congruent to b modulo n).

Exercises

Exercise 1. A variant of Exercise 3.2.1d. p112

Theorem 1. The integer n is odd if and only if n^3 is odd.

ER 1i. Sybolically write Theorem 1.

ER 1ii. Prove Theorem 1. You may use any result in §1.3 (Ch. 1 Summary p. 31-32) and/or the definition of even/odd. You may not use Exercise 3.2.1 a,b, or c.

Exercise 2. A variant of Exercise 3.2.5. p112

Conjecture 2. For all integers a and b, if ab is even, then a is even or b is even.

ER 2i. Sybolically write Conjecture 2.

ER 2ii. Determine if Conjecture 2 is true or false. If Conjecture 2 is true, then write a formal proof of Conjecture 2. If Conjecture 2 is false, then provide a counterexample that shows (and clearly explains) why Conjecture 2 if false.

Exercise 3. A variant of Exercise 3.2.7. p113

Conjecture 3. For each integer a, we have that $a \equiv\ 2 \pmod 8$ if and only if $(a^2 + 4a) \equiv\ 4 \pmod 8$.

ER 3i. Sybolically write Conjecture 3, using the biconditional ($\Longleftrightarrow$). Next sybolically write Conjecture 3 as the conjection ($\wedge$) of two conditional ($\Longrightarrow$) statements.

ER 3ii. For each of the two conditional statements in Part (3i), determine if the conditional statement is true or false. If the conditional statement is true, write a proof. If it is false, provide a counterexample.

ER 3ii. Is Conjecture 3 true or false? Explain.

Exercise 4. A variant of Exercise 3.2.16. p114

Let y_1, y_2, y_3, y_4 be real numbers. The **mean**, $\overline{y}$, of these four numbers is defined to be the sum of the four numbers divided by 4. That is,

$$\overline{y} = \frac{y_1 + y_2 + y_3 + y_4}{4}.$$

Prove that there exists a y_i with $1 \le i \le 4$ such that $\overline{y} \le y_i$.

Hint: One way is to let $y_{\max}$ be the largest of y_1, y_2, y_3, y_4 .