LaTex Help

Let $n \in \mathbb{Z}^{\neq 0}$ and $a, b \in \mathbb{Z}$.

▷ The notation for n divides a is $n|a$.

▷ The notation for n does not divide a is $n \nmid a$.

▷ The notation a is congruent to b modulo n is $a \equiv b \pmod{n}$.

Look at the above LaTex to see how the math symbols are easily done in LaTeX.

Warning

Henceforth, when asked to *symbolically write* a statement, do so **using quanifiers**.

Definition for Congruence

In class we showed the following.

Let $n \in \mathbb{N}$ and $a, b \in \mathbb{Z}$. The following 10 statements are equivalent (TFAE).

(1) a is congruent to b modulo n

(2) $a \equiv b \pmod{n}$

(3) n divides $a - b$

(4) $(\exists k \in \mathbb{Z})\,[a - b = nk]$

(5) $(\exists k \in \mathbb{Z})\,[a = b + nk]$

(6) b is congruent to a modulo n

(7) $b \equiv a \pmod{n}$

(8) n divides $b - a$

(9) $(\exists j \in \mathbb{Z})\,[b - a = nj]$ to see (4)⇔(9): $j = -k$

(10) $(\exists j \in \mathbb{Z})\,[b = a + nj]$

The equivalence of (1)–(5) are just notation or follow directly from definitions or algebra.

Similarly, the equivalence of (6)–(10) are just notation or follow directly from definitions or algebra.

To see that (4) is equivalent to (9), let $j = -k$. Thus (1) – (10) are indeed all equivalent.

▶ On homework and exams, as the definition of a is congruent to b modulo n your may use either: (3), (4), (5), (8), (9), or (10).

Exercises

Exercise 1. A variant of Exercise 3.1.9b. p97

Theorem 1. For integers a and b, if $a \equiv 7 \pmod{8}$ and $b \equiv 3 \pmod{8}$, then $a \cdot b \equiv 5 \pmod{8}$.

ER 1i. Sybolically write Theorem 1. As universes, use $\mathbb{Z}$ and/or $\mathbb{Z}^{\neq 0}$ and/or some cross product of these.

ER 1ii. Prove Theorem 1 using the definition of congruence (as indicated above in the TFAE). To be clear, you may not use Exercise 3.1.12 (p. 98) nor facts from the Congruence Group Work. A proof which basically says that Theorem 1 is immediate from modulo arithmetic will receive zero points. Do use the proofs from these sources to help guide you through your proof of Theorem 1, which is basically asking you to verify modulo arithmetic holds for some given specific numbers. tt

Exercise 2. A variant of Exercise 3.1.16c. p99

Conjecture 2. For nonnegative real numbers x and y

$$\sqrt{xy} \leq \frac{x+y}{2}.$$

ER 2i. Sybolically write Conjecture 2. As universes, use $\mathbb{R}^{\geq 0}$ and/or some cross product of these.

ER 2ii. Determine if Conjecture 2 is true or false. If Conjecture 2 is true, then write a formal proof of Conjecture 2. If Conjecture 2 is false, then provide a counterexample that shows (and clearly explains) why Conjecture 2 if false. If the conjecture is true and you write a proof, you may not use previous shown result in Exercise 3.1.6 b (p. 99, look at it!), which we proved in class (however the proof of this previous result might help you).

Exercise 3. A variant of Exercise 3.1.21a. p102

Definitions. Three natural numbers a, b, and c with $a < b < c$ are called a Pythagorean triple provided that $a^2 + b^2 = c^2$. See Exercise (13) on page 30 in Section 1.2. Three natural numbers are called consecutive natural numbers if they can be written in the m, $m+1$, and $m+2$.

Theorem 3. The only Pythagorean triple consisting of three consecutive numbers is 3,4,5.

ER 3i. Sybolically write Theorem 3. As universes $\mathbb{N}^3$. Hint: recall that $\exists!$ denotes *there exists a unique*.

ER 3ii. Prove Theorem 3. Hint: got to show existence and uniqueness.