LaTex Help

Def. A nonzero integer m divides an integer n, denoted $m|n$, provided that $(\exists q \in \mathbb{Z})\ [qm = n]$. p82

Remark. The notation for a $m \in \mathbb{Z}^{\neq 0}$ not dividing $n \in \mathbb{Z}$ is $m \nmid n$.

Warning

Henceforth, when asked to *symbolically write* a statement, do so **using quanifiers**.

Exercises

Exercise 1. A variant of Exercise 3.1.3c. p96

Conjecture 1. For all integers a, b, and c with $a \neq 0$, if a divides $b-1$ and a divides $c-1$, then a divides $bc-1$.

ER 1i. Sybolically write Conjecture 1. As universes, use $\mathbb{Z}$ and/or $\mathbb{Z}^{\neq 0}$ and/or some cross product of these.

Hint: don't forget needed parentheses, e.g., $a|b-1$ does not make sense and should be written as $a|\,(b-1)$.

ER 1ii. Determine if Conjecture 1 is true or false. If Conjecture 1 is true, then write a formal proof of Conjecture 1. If Conjecture 1 is false, then provide a counterexample that shows (and clearly explains) why Conjecture 1 if false.

Exercise 2. A variant of Exercise 3.1.3h. p96

Conjecture 2. For all integers a, b, and c with $a \neq 0$, if a divides bc then a divides b or a divides c.

ER 2i. Sybolically write Conjecture 2. As universes, use $\mathbb{Z}$ and/or $\mathbb{Z}^{\neq 0}$ and/or some cross product of these.

ER 2ii. Determine if Conjecture 2 is true or false. If Conjecture 2 is true, then write a formal proof of Conjecture 2. If Conjecture 2 is false, then provide a counterexample that shows (and clearly explains) why Conjecture 2 if false.

Exercise 3. A variant of Exercise 3.1.6b. p97

Theorem 3. For each integer a, if there exists an integer n such that a divides $9n+5$ and a divides $6n+1$, then a divides 7.

ER 3i. Sybolically written, Conjecture 3 is

$$(\forall a \in \mathbb{Z})\ \Big[\ (\exists n \in \mathbb{Z})\,[\,a|\,(9n+5) \wedge a|\,(6n+1)] \implies a|7\ \Big]. \tag{3s}$$

In attempts to simplify (3s) (so we can see better what is going on), we let

$P(a)$ be the open sentence $(\exists n \in \mathbb{Z})\,[\,a|\,(9n+5) \wedge a|\,(6n+1)]$

$Q(a)$ be the open sentence $a|7$

$R(a)$ be the open sentence $P(a) \implies Q(a)$.

Then (3s) can be expressed as

$$(\forall a \in \mathbb{Z}) \ [\ P(a) \implies Q(a) \ ] \quad \text{as well as simply just} \quad (\forall a \in \mathbb{Z}) \ [\ R(a) \ ] \, .$$

Explain why $R(0)$ is true. (If you are confused about the notation, see Example 2.10 on page 57.)

ER 3ii. Prove Theorem 3.