Covers sections: 2.2, 2.3, and 2.4.

Exercise 1. A variant of Exercise 2.2.3fg.

Write a useful negation of each of the following statements.

⟨Remember, we discussed *useful* in class (e.g., do not talk in double negatives.⟩

Exercise 1f. If you graduate from UofSC, then you will get a job or you will go to graduate school.

Exercise 1g. If I play tennis, then I will wash the car or I will do the dishes.

Exercise 2. A variant of Exercise 2.2.9e.

Use previously proven logical equivalencies (e.g., 1-12 from the §2.2 Handout) to prove the following logical equivalency. Clearly justify (e.g., say which logical equivalency your are using) your steps.

$$[\,(P \implies Q) \implies R\,] \equiv [\,(\sim P \implies R) \wedge (Q \implies R)\,]$$

Exercise 3a. Exercise 2.3.4d.

Use the roster method to specify the truth set for the following open sentence $P(n)$. The universal set for each open sentence is the set of integers $\mathbb{Z}$.

$$n \text{ is an odd integer that is greater than 2 and less than 14.}$$

Exercise 3b. A variant of Exercise 2.4.2e.

For the following statement, use a counterexample to show that the statement is false.

$$(\forall a \in \mathbb{Z}) \left[\sqrt{a^2} = a\right].$$

Exercise 4. A variant of Exercise 2.4.3d.

Consider the following statement.

$$(\exists x \in \mathbb{Q}) \left[\ \sqrt{2} < x < \sqrt{3}\ \right].$$

Note: The sentence "$\sqrt{2} < x < \sqrt{3}$" is actually a conjuction. It means "$\sqrt{2} < x$" and "$x < \sqrt{3}$".

Exercise 4i. Write the statement as an English sentence that does not use the symbols for quantifiers. ⟨See Writing Guideline number 12 to see that it is fine to use symbols such as: +. =, <.⟩

Exercise 4ii. Write the negation of the statement in symbolic form (i.e., symbolically) in which the negation symbol (i.e., $\sim$) is not used.

Exercise 4iii. Write a useful negation of the statement in an English sentence that does not use the symbols for quantifiers.

Exercise 5. A variant of Exercise 2.4.14aef.

Let A be a subset of the real numbers. A number $b \in \mathbb{R}$ is called an upper bound for the set A provided that for each element $x \in A$, we have $x \le b$.

Exercise 5a. Write this definition in symbolic form by completing the following: Let A be a subset of the real numbers. A number b is called an upper bound for the set A provided that . . . .

Exercise 5e. Complete the following in symbolic form: Let A be a subset of $\mathbb{R}$. A number b is not an upper bound for the set A provided that . . . .

Exercise 5f. Without using the symbols for quantifiers, complete the following sentence: Let A be a subset of $\mathbb{R}$. A number b is not an upper bound for the set A provided that . . . .