Previous Results

Exercise 1.2.1a. If m is an even integer, then $m + 1$ is an odd integer.

Exercise 1.2.1b. If m is an odd integer, then $m + 1$ is an even integer.

Exercise 1.2.2a. If x is an even integer and y is an even integer, then $x + y$ is an even integer.

Exercise 1.2.2b. If x is an even integer and y is an odd integer, then $x + y$ is an odd integer.

Exercise 1.2.2c. If x is an odd integer and y is an odd integer, then $x + y$ is an even integer.

Exercise 1.2.3a. If x is an even integer and y is an integer, then $x \cdot y$ is an even integer.

Theorem 1.8. If x is an odd integer and y is an odd integer, then $x \cdot y$ is an odd integer.

Exercise 1. A variant of Exercise 1.2.5a.

Prove the below Theorem A using the definitions of even integer and odd integer.

You may not use the Previous Results stated above.

Theorem A. If m is an even integer, then $3m^2 + 2m + 3$ is an odd integer.

Proof. Put your proof here. ■

Exercise 2. A variant of Exercise 1.2.5a.

Prove Theorem A from the previous Exercise by using the Previous Results stated above rather than using the definitions of even integer and odd integer. You may use the parity (i.e., the fact of being even or odd) of a specific given fixed integer (i.e., 1 is odd, 2 is even, 3 is odd, ...).

Proof. Put your proof here. ■

Exercise 3. A variant of Exercise 1.2.7a.

Is the below Conjecture 3 true or false? Justify your answer.

You should understand (and this will not be repeated) that the instructions means

- if Conjecture 3 is false, then say Conjecture 3 is false and then justify by providing a counterexample.
- if Conjecture 3 is true, then say Conjecture 3 is true and then justify by providing a proof.

Conjecture 3. If a, b, and c are integers, then $ab + ac$ is an even integer.

Exercise 4. A variant of Exercise 1.2.7b.

Is Conjecture 4 true or false? Justify your answer.

Conjecture 4. If b and c are odd integers and a is an integer, then $ab + ac$ is an even integer.