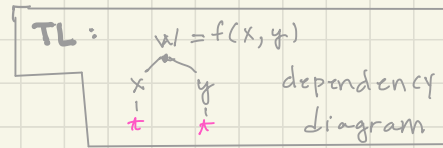


14.4 Chain Rule (CR)

diff = differentiable

HD 14.4.1

Thm 1 Let $w = f(x, y)$ and $x = x(t)$ and $y = y(t)$ be differentiable.



Then $w = f(x(t), y(t))$ is a diff. function of t and

$$\frac{dw}{dt}(t) = f_x(x(t), y(t)) x'(t) + f_y(x(t), y(t)) y'(t)$$

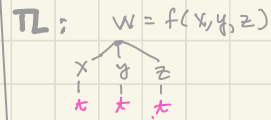
in short

$$\frac{dw}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt}$$

i.e.:

$$\frac{dw}{dt} = \frac{\partial w}{\partial x} \frac{dx}{dt} + \frac{\partial w}{\partial y} \frac{dy}{dt}$$

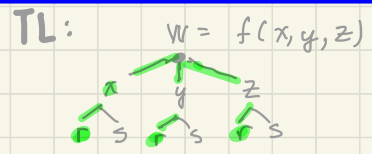
Thm 2 Let $w = f(x, y, z)$ and $x = x(t)$ and $y = y(t)$ and $z = z(t)$ be differentiable.



Then $w = f(x(t), y(t), z(t))$ is a diff. function of t and

$$\frac{dw}{dt} = \frac{\partial w}{\partial x} \frac{dx}{dt} + \frac{\partial w}{\partial y} \frac{dy}{dt} + \frac{\partial w}{\partial z} \frac{dz}{dt}$$

Thm 3 Let $w = f(x, y, z)$ and $x = x(r, s)$ and $y = y(r, s)$ and $z = z(r, s)$ be diff.



Then $w = f(x(r, s), y(r, s), z(r, s))$ has partial derivatives w.r.t r and s and

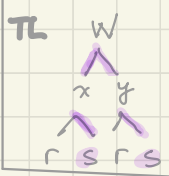
$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial r} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial r} \quad \text{AND} \quad \frac{\partial w}{\partial s} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial s} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial s}$$

Thm 4 Let $w = f(x, y)$ and $x = x(r, s)$ and $y = y(r, s)$ be diff.

Then $w = f(x(r, s), y(r, s))$ has partial derivatives wrt r and s .

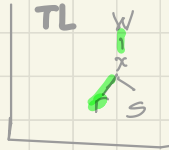
and

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial r} \quad \text{AND} \quad \frac{\partial w}{\partial s} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial s}$$



Thm 5 Let $w = f(x)$ and $x = x(r, s)$ be diff.

Then $w = f(x(r, s))$ has partial derivatives w.r.t r and s and



$$\frac{\partial w}{\partial r} = \frac{dw}{dx} \frac{\partial x}{\partial r} \quad \text{AND} \quad \frac{\partial w}{\partial s} = \frac{dw}{dx} \frac{\partial x}{\partial s}$$

Implicit Differentiation

Thm If $F(x, y)$ is diff. and
the equation $F(x, y) = 0$ defines a diff. function of x
then

$$\frac{dy}{dx} = - \frac{F_x}{F_y}$$

at any point where $F_y \neq 0$.



So $F(x, y) = 0$ defines some diff. function $y = g(x)$.
Luckily, we do not have to find the function g .

Thm If $F(x, y, z)$ is diff. and
the equation $F(x, y, z) = 0$ defines a diff. function of x and y

then

$$\frac{\partial z}{\partial x} = - \frac{F_x}{F_z} \quad \text{and} \quad \frac{\partial z}{\partial y} = - \frac{F_y}{F_z}$$

at any point where $F_z \neq 0$.



So $F(x, y, z) = 0$ defines some diff. function $z = g(x, y)$.
Luckily, we do not have to find the function g .