

Ex 1. Let

$$f(x, y) = \sqrt{9 - x^2 - y^2}.$$

1.1. Find $\frac{\partial f}{\partial y}(x, y)$. TL: We are differentiating w.r.t. y so we treat as constant the variable x .

$$\begin{aligned} \frac{\partial}{\partial y} (9 - x^2 - y^2)^{1/2} &\stackrel{\text{chain rule}}{=} \frac{1}{2}(9 - x^2 - y^2)^{-1/2} \left(\frac{\partial}{\partial y}(9 - x^2 - y^2) \right) \\ &= \frac{1}{2}(9 - x^2 - y^2)^{-1/2}(-2y) \\ &= \boxed{\frac{-y}{\sqrt{9 - x^2 - y^2}}}. \end{aligned}$$

1.2. Find $f_y(1, 2)$.

$$f_y(1, 2) \stackrel{\text{notation}}{=} \frac{\partial f}{\partial y}(1, 2) = \frac{-2}{\sqrt{9 - 1^2 - 2^2}} = \frac{-2}{\sqrt{9 - 1 - 4}} = \frac{-2}{\sqrt{4}} = \boxed{-1}.$$

Ex 2. Do Desmos Demo 14.3.1, which goes with Example 1.

Ex 3. Find $\frac{\partial}{\partial z}(ze^{xyz})$.

$$\text{Soln. } \frac{\partial}{\partial z}(ze^{xyz}) =$$

$$\begin{aligned} \frac{\partial}{\partial z}(ze^{xyz}) &\stackrel{\text{product rule}}{=} \left(\frac{\partial}{\partial z} z \right) e^{xyz} + z \left(\frac{\partial}{\partial z} e^{xyz} \right) \\ &\stackrel{\text{chain rule}}{=} 1 \cdot e^{xyz} + ze^{xyz} \left(\frac{\partial}{\partial z}(xyz) \right) \\ &= e^{xyz} + ze^{xyz}(xy) \\ &= \boxed{e^{xyz}(1 + xyz)}. \end{aligned}$$

Ex 4. Find all the second partial derivatives of

$$f(x, y) = y^3 + 5y^2e^{4x} - \cos(x^2).$$

First we need the first-order partial derivatives:

$$\begin{aligned} f_x(x, y) &= 0 + 5y^2e^{4x}(4) - [-\sin(x^2)](2x) \\ &= 20y^2e^{4x} + 2x\sin(x^2), \end{aligned}$$

$$f_y(x, y) = 3y^2 + 10ye^{4x}.$$

Now we can find the second-order partial derivatives.

Not mixed:

$$\begin{aligned} f_{xx}(x, y) &= (20y^2)(e^{4x}(4)) + [2\sin(x^2) + (2x)\cos(x^2)](2x) \\ &= \boxed{80y^2e^{4x} + 2\sin(x^2) + 4x^2\cos(x^2)}, \end{aligned}$$

$$f_{yy}(x, y) = \boxed{6y + 10e^{4x}}.$$

Mixed:

$$f_{xy}(x, y) \stackrel{\text{i.e.}}{=} \frac{\partial}{\partial y} f_x(x, y) = \frac{\partial}{\partial y} (20y^2e^{4x} + 2x\sin(x^2)) = 40ye^{4x} + 0 = \boxed{40ye^{4x}},$$

$$f_{yx}(x, y) \stackrel{\text{i.e.}}{=} \frac{\partial}{\partial x} f_y(x, y) = \frac{\partial}{\partial x} (3y^2 + 10ye^{4x}) = 0 + (10y) \frac{\partial}{\partial x} e^{4x} = (10y)(e^{4x}) \left(\frac{\partial}{\partial x} (4x) \right) = \boxed{40ye^{4x}}.$$

Note. The mixed partial derivatives are equal: $f_{xy} = f_{yx}$.

Ex 5. Find f_{zyxx} for $f: \mathbb{R}^3 \rightarrow \mathbb{R}$ with

$$f(x, y, z) = x(z^2 + ye^z)^3.$$