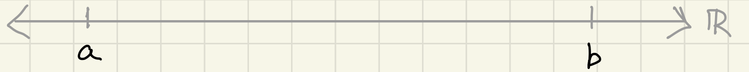
14.1 Part B. Open and Friend

Before giving definitions, let's see how far our intuition takes us.

Ex 1 in $\mathbb{R}^1$. Let $D^1 = [a,b) \overset{\text{i.e.}}{=} \{x \in \mathbb{R}^1 : a \leq x < b\}$.

1.1 Sketch D^1.

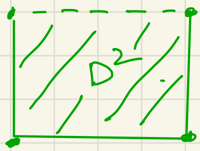


1.2 interior of D^1 = ____________________.

1.3 boundary of D^1 = ____________________.

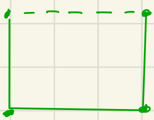
1.4 Is D^1 open? Yes NO. Is D^1 closed? Yes No.

Ex 2 in $\mathbb{R}^2$.

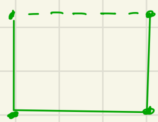


$\leftarrow D^2$ is the "inside" of a square in $\mathbb{R}^2$, not including the square's "top" but including its other 3 sides.

2.1 Indicate the interior of D^2



2.2 Indicate the Boundary of D^2



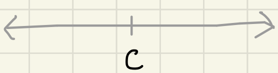
Def 1 Let $c \in \mathbb{R}^n$ and $r > 0$. The $\underbrace{\text{r-neighborhood}}_{}$ about the point c is the set (NBHD)

$N_r(c)$ = the set of $p \in \mathbb{R}^n$ who distance from c is strictly less than r.

$\overset{\text{so}}{=} \{p \in \mathbb{R}^n : d(c,p) < r\}$.

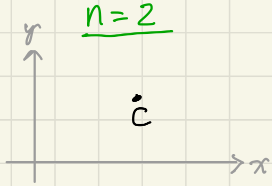
Ex 3 Sketch $N_r(c)$ for $n = 1, 2, 3$.

$n=1$



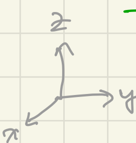
$N_r(c) =$

$n=2$



$N_r(c)$ is

$n=3$



$\dot{c}$

$N_r(c)$ is

For each $n = 1, 2, 3$.

$N_r(c)$

c is center

r is radius

$\boxed{\text{Defs}}$

▶ Let $D^n \subseteq \mathbb{R}^n$ and $c \in \mathbb{R}^n$

"a subset of" "an element of" ← Recall Set Notation

Def 1 Let $c \in \mathbb{R}^n$ and $r > 0$. The <u>r-neighborhood about the point c</u> is the set

$N_r(c)$ = the set of $p \in \mathbb{R}^n$ who distance from c is strictly less than r.

$$\stackrel{\text{so}}{=} \{ p \in \mathbb{R}^n : d(p, c) < r \}.$$

Def 2a c is an <u>interior point</u> of D^n $\Leftrightarrow$ we can find $r > 0$ with $N_r(c) \subseteq D^n$.

"inside"

think of taking <u>small</u> $r > 0$

Def 2b The set of all interior pts of D^n is the <u>interior of D^n</u>

Ex 4 revisit Ex 2.

Def 3a c is a <u>boundary pt</u> of D^n $\Leftrightarrow$ <u>each</u> $N_r(c)$ contain a pt in D^n and also a point not in D^n.

Def 3b The set of all boundary points of D^n is the <u>boundary</u> of D^n.

Ex 5 revisit Ex 2.

Def 4a D^n is <u>open</u> $\Leftrightarrow$ each pt. in D^n is an interior of D^n.

Def 4b D^n is <u>closed</u> $\Leftrightarrow$ D^n contains all it's boundary points.

Ex 6 Is Ex2's D^2 open? ________. Is Ex2's D^2 closed? ________

Def 5a D^n is bounded $\stackrel{\text{def}}{\Leftrightarrow}$ D^n is contained in some neighborhood,

$\stackrel{\text{i.e.}}{\Leftrightarrow}$ we can find $r > 0$ and $c \in \mathbb{R}^n$ with $D^n \subseteq N_r(c)$

think of taking <u>large</u> $r > 0$ (and c can be any pt.)

Def 5b D^n is unbounded $\Leftrightarrow$ D^n is not bounded.

Ex 7. Is Ex2's D^2 bounded or unbounded?