

From Ch 13 to Ch 14.

Ch 13 $\rightarrow$ 14.

Now:

$$\mathbb{D}^n \subseteq \mathbb{R}^n$$

($\mathbb{D}^n$ will be the Domain of a function).

Ch 13 had $\vec{F} : \mathbb{D}^1 \rightarrow \mathbb{R}^m$ $\leftarrow$ paths
Ch 14 has $f : \mathbb{D}^n \rightarrow \mathbb{R}^1$ $\leftarrow$ function of n -variables } $m, n = 2 \text{ or } 3$ usually

Recall The trace on a surface S by a plane P is $S \cap P$

Given $f : \mathbb{D}^2 \rightarrow \mathbb{R}^1$ with $z = f(x, y)$ and a constant $c \in \mathbb{R}$.

Get a surface $S = \{ (x, y, f(x, y)) : (x, y) \in \mathbb{D}^2 \} \subseteq \mathbb{R}^3$

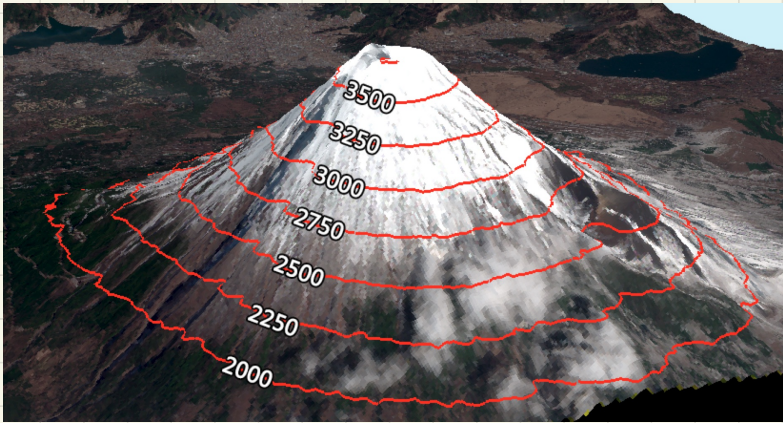
- The trace on S by plane $x = c$ is $\{ (c, y, f(c, y)) : (c, y) \in \mathbb{D}^2 \} \stackrel{\text{note}}{\subseteq} \mathbb{R}^3$
- The trace on S by plane $y = c$ is $\{ (x, c, f(x, c)) : (x, c) \in \mathbb{D}^2 \} \subseteq \mathbb{R}^3$
- The trace on S by plane $\boxed{z = c}$ is
 $\{ (x, y, f(x, y)) : f(x, y) = c \text{ and } (x, y) \in \mathbb{D}^2 \} \subseteq \mathbb{R}^3 \leftarrow xy\text{-}z\text{-space}$

Def The level set of value c is $\{ (x, y) \in \mathbb{D}^2 : f(x, y) = c \} \subseteq \mathbb{R}^2$
 $xy\text{-plane}$

Ex of level sets is a topographic map where
 $f : \mathbb{D}^2 \rightarrow \mathbb{R}$ and $f(x, y) = \text{height of the mountain at } (x, y)$
note: $\{ (x, y, f(x, y)) : (x, y) \in \mathbb{D}^2 \}$ is the surface of the mountain

Mount Fuji

is our surface S .

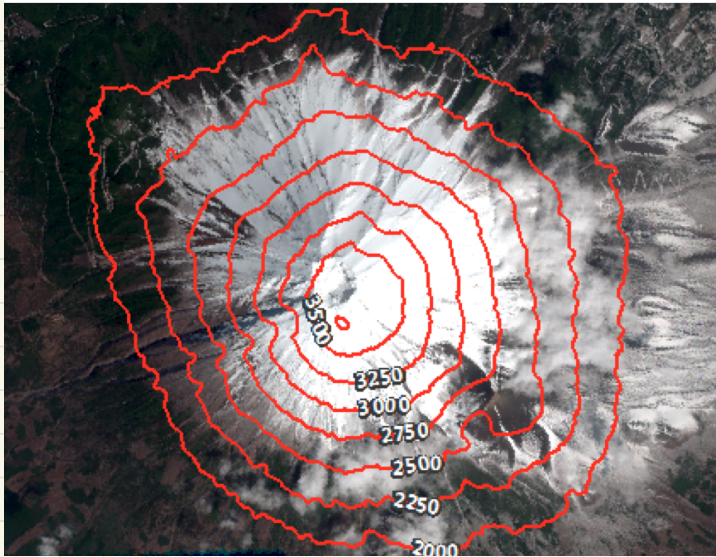


traces

by planes $z=c$

in xyz-space

3D



level sets

of value c

in xy-space.

2D

(topographic map)

Rmk level sets are in xy -plane ... think of as:

- lift level set up to surface to get trace by $z=c$

or

- push trace by $z=c$ down to xy -plane to get level set
 $\xrightarrow{I_3}$