§ 13.2 Integrals of paths

Setting

- Given a path $\boxed{\vec{F} : \mathbb{D}^1 \to \mathbb{V}^m}$ (usually $m=2$ or $m=3$)

  domain $\mathbb{D}^1$ is interval of $\mathbb{R}^1$ ↵ ↳ set of all vectors in $\mathbb{R}^m$

  So $\vec{F}(t) = \langle f_1(t), f_2(t), \ldots, f_m(t) \rangle$ where each $\underbrace{f_i : \mathbb{D}^1 \to \mathbb{R}^1}$.

  note: f_i is $\mathbb{R}^1$-valued so can use Calc. 1 & 2 methods on f_i

- Given an interval $[a,b] \subseteq \mathbb{D}^1 \subseteq \mathbb{R}^1$
- Denote an arbitrary constant vector $\vec{C} \overset{\text{by}}{=} \langle c_1, c_2, \ldots, c_m \rangle \in \mathbb{V}^m$ so $c_i \in \mathbb{R}$

Def Definite Integral of $\vec{F}$ over $[a,b]$

$$\int_a^b \vec{F}(t)\,dt \underset{\text{also written}}{=} \int_a^b \langle f_1(t), f_2(t), \ldots, f_m(t) \rangle\,dt \underset{\text{by def.}}{=} \left\langle \int_a^b f_1(t)\,dt, \int_a^b f_2(t)\,dt, \ldots, \int_a^b f_m(t)\,dt \right\rangle.$$

Def Indefinite Integral of $\vec{F} : \mathbb{D}^1 \to \mathbb{V}^m$ is $\vec{G} : \mathbb{D}^1 \to \mathbb{V}^m$,

denoted $\int \vec{F}(t)\,dt = \vec{G}(t) + \vec{C}$,

$\Longleftrightarrow$ $\vec{F} = \vec{G}'$ on $\mathbb{D}^1$.

Ex 1. $\int \langle 1, 2t \rangle\,dt = \langle \_\_\_, \_\_\_ \rangle$ $\underset{\text{ok}}{\text{also}}$ $\langle \_, \_ \rangle + \vec{C}$

Soln $\int \langle 1, 2t \rangle\,dt =$

Ex 2 $\int_5^6 \langle 1, 2t \rangle\,dt = \langle \_\_, \_\_ \rangle$

| Way 1 Use Ex 1 | $\int_5^6 \langle 1, 2t \rangle\,dt =$

| Way 2 w/o Ex | $\int_5^6 \langle 1, 2t \rangle\,dt =$

Know   More on Motion   So   13.2.2

$\frac{d}{dt}$ $\vec{r}(t)$ position vector $\int$ wrt t

$\frac{d}{dt}$ $\vec{v}(t)$ velocity vector $\int$ wrt t

$\vec{a}(t)$ acceleration vector

So: $\int \vec{v}(t)\,dt = \int \vec{r}\,'(t)\,dt = \vec{r}(t) + \vec{C}$ and $\int \vec{a}(t)\,dt = \int \vec{v}\,'(t)\,dt = \vec{v}(t) + \vec{K}$.

Ex 3 A puffo is moving along a curve with this

(a) $\vec{a}(t) = \langle 0, 2, 9e^{3t} \rangle$ ← recall $\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t)$

(b) $\vec{v}(0) = \langle 5, 0, 3 \rangle$ ← recall $\vec{v}(t) = \vec{r}\,'(t)$

(c) $\vec{r}(1) = \langle 5, 8, e^3 \rangle$.

His position vector is (fill in the blanks)

$$\vec{r}(t) = \langle \_\_\_\_\_\_, \_\_\_\_\_\_, \_\_\_\_\_\_ \rangle$$

Ex 3 (reworded) Solve the differential equation $\frac{d^2\vec{r}(t)}{dt^2} = \langle 0, 2, 9e^{3t} \rangle$ that satisfies $\vec{r}\,'(t) = \langle 5, 0, 3 \rangle$ and $\vec{r}(1) = \langle 5, 8, e^3 \rangle$.

Soln