§ 12.6 Cylinders and Quadric Surfaces

Recall

§ 12.1

Def The trace on a surface $\mathcal{S}$ by another surface $\mathcal{P}$ (often a plane) is: the set of points on $\mathcal{S}$ that are also on $\mathcal{P}$.

Note 1. So trace on $\mathcal{S}$ by $\mathcal{P}$ is $\mathcal{S} \cap \mathcal{P}$. ← $\cap$ is for intersect.

2. can think of trace "sits" on $\mathcal{S}$ in $\mathbb{R}^3$.

Ex trace on a tennis ball by a plane is a circle.

Ex Another type of surface is a cylinder in $\mathbb{R}^3$ that is generated by an equation of 2 variables (e.g. $x^2+y^2=4$... so Mr. z can be anything)

In § 12.1 we did Desmos 12.6.1, which has 4 examples of cylinders.

Ex The trace of the cylinder $x^2+y^4=4$ by the plane $z=my$ is: ellipse.

Recall this was Desmos Demo 12.1.4 = paper towel roll.

§ 14.1

Ex Another type of surface is an ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

See class notes § 14.1 and revisit Desmos Demo 12.6.3.

Rmk A trace by a plane is also called a cross-section.

Surfaces $\mathcal{S}$ in $\mathbb{R}^3$ that are given by equation in 3 variables.

- Examples of surfaces we have studied (for 3 variables: x, y, z)
  1. plane ($ax+by+cz=d$)
  2. sphere ($x^2+y^2+z^2=r^2$)
  3. cylinder (e.g. $x^2+y^2=4$)
  4. ellipsoid ($\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$)

- New surface for constants $a>0$, $b>0$, $c>0$ and variables: V_1, V_2, V_3:

$$\frac{(V_1)^2}{a^2} - \frac{(V_2)^2}{b^2} = \frac{(V_3)^1}{c^1}$$

← 1, not 2

← c^1, not c^2.

is a

Hyperbolic Paraboloid (saddle).

Hyperbolic Paraboloid (saddle)

for constants $a>0$, $b>0$, $c>0$ and

$$\boxed{\frac{y^2}{b^2}-\frac{x^2}{a^2}=\frac{z}{c}}.$$

← Note, c not c^2.

So $z = \frac{c}{b^2}y^2 - \frac{c}{a^2}x^2$.

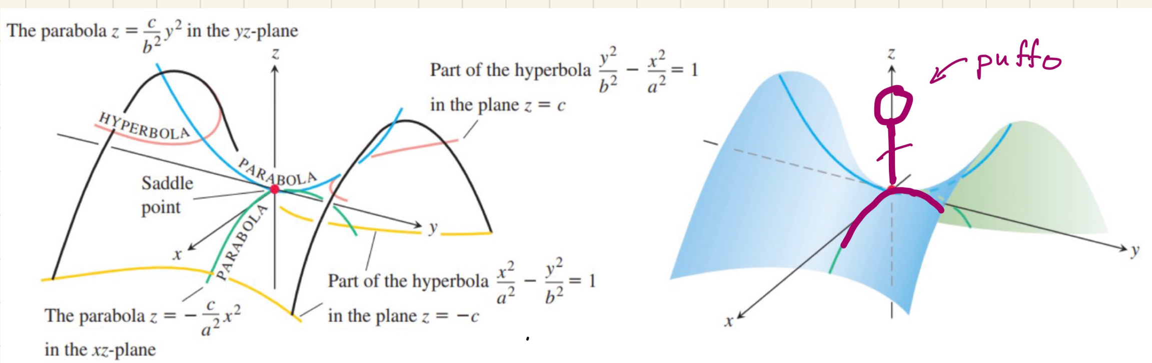
1. Trace by the plane $x=0$ is $z=\frac{c}{b^2}y^2$
   Trace by the plane $x=X_0$ is $z=\frac{c}{b^2}y^2 - \underbrace{\frac{cX_0^2}{a^2}}_{\text{number}}$ } parabola CCU

2. Trace by the plane $y=0$ is $z=-\frac{c}{a^2}x^2$
   Trace by the plane $y=y_0$ is $z=-\frac{c}{a^2}x^2 + \overbrace{\frac{cy_0^2}{b^2}}^{\text{number}}$ } parabola CCD

3. Trace by the plane $z=0$ is $\frac{y^2}{b^2}-\frac{x^2}{a^2}=\frac{0}{c} \Rightarrow y^2=\frac{b^2}{a^2}x^2 \Rightarrow y=\pm\frac{b}{a}x$
   So trace by $z=0$ is the 2 lines $y=\pm\frac{b}{a}x$ thru $(0,0)$.
   Trace by the plane $z=Z_0$ when $z_0\neq 0$ is (do the algebra)
   $\frac{y^2}{b^2}-\frac{x^2}{a^2}=\frac{z_0}{c} \overset{(A)}{\Rightarrow} \frac{c}{z_0b^2}y^2-\frac{c}{z_0a^2}x^2=1$ ← hyperbola.

- The saddle is symmetric abt the planes $x=0$ and $y=0$. (no change if replace x w/ $-x$).

A (color-coded) sketch.



The saddle point is $(0,0,0)$.

* If a puffo is sitting on the saddle $\frac{y^2}{b^2}-\frac{x^2}{a^2}=\frac{z}{c}$ then:
  - his legs are hanging over the CCD parabola $z=-\frac{c}{a^2}x^2$ in the $\underbrace{\text{plane } y=0}_{xz\text{-plane}}$
  - rest of is body is on positive z-axis.

- Do Desmos Demo 12.6 on Hyperbolic Paraboloid (saddle).

- See version on our Course Handout page for next 2 pages

See version on our Course Handout page for this and next page.

Six Quadric Surfaces in § 12.6

Six Quadric Surfaces

- Ellipsoid and Hyperbolic Paraboloid (saddle).
    - We discussed in class and there are Desmos Demos.
    - Know well and understand
    - Know by name.

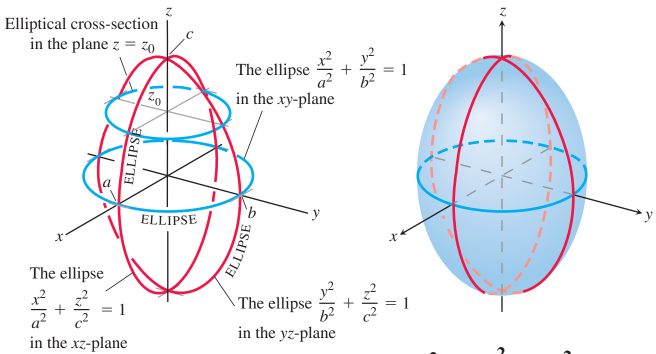
- Elliptical Paraboloid and Elliptical cone.
    - Will be used for in class as examples so understand well enough to follow a class example with these 2 elliptic surfaces.
    - Do not need to know by name.

- Hyperboloid of 1 sheet and Hyperboloid of 2 sheet
    - Will not use. Will not covered.
    - You do not need to know.

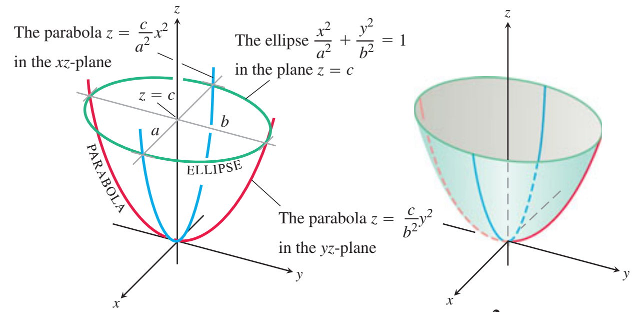
Sketchs / Summary of these 6 Quadric Surfaces.

See next page, which is from our book (Thomas 15th ed), p.752.

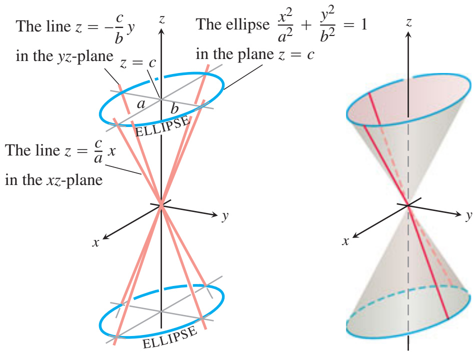
TABLE 12.1 Graphs of Quadric Surfaces



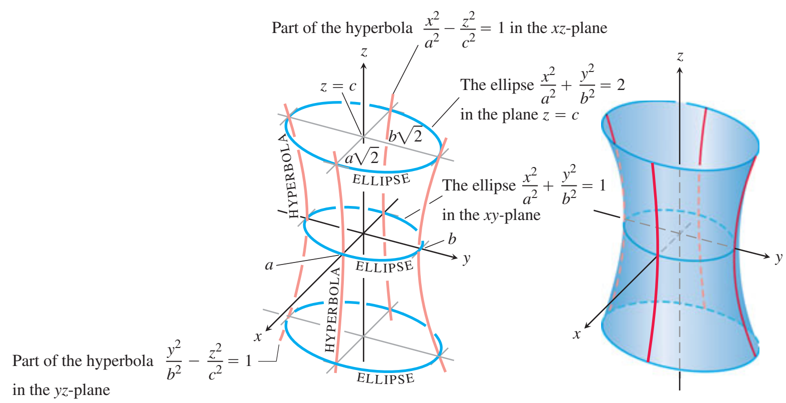
ELLIPSOID $\quad \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$



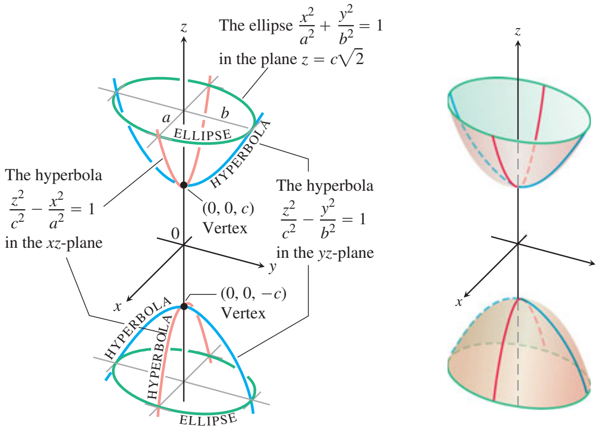
ELLIPTICAL PARABOLOID $\quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$



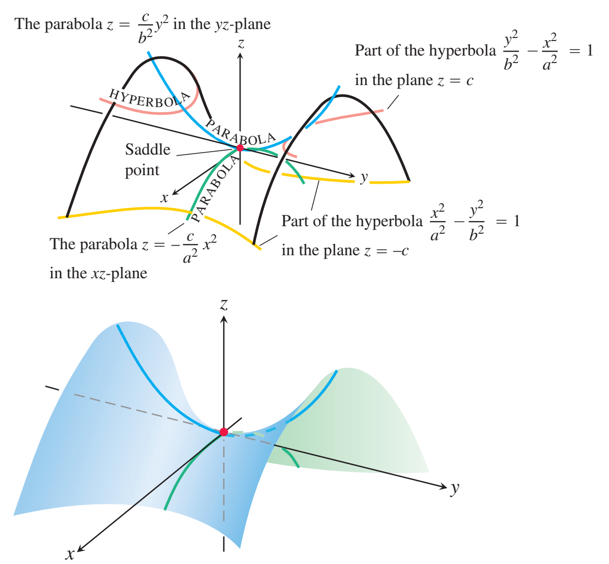
ELLIPTICAL CONE $\quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$



HYPERBOLOID OF ONE SHEET $\quad \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$



HYPERBOLOID OF TWO SHEETS $\quad \frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$



HYPERBOLIC PARABOLOID $\quad \frac{y^2}{b^2} - \frac{x^2}{a^2} = \frac{z}{c}, c > 0$

(saddle)