§ 12.6 Cylinders and Quadric Surfaces

Cylinders

▶ We covered cylinders in § 12.1.
Recall the below class notes from § 12.1.

Def From § 12.6. A cylinder $\mathcal{S}$ ← a fancy S.
In $\mathbb{R}^3$, start with :
and
- a generating curve $\mathcal{C}$ in a plane $\mathcal{P}$
- a generating line $\mathcal{L}$ that intersects $\mathcal{P}$ in exactly 1 point.

The cylinder $\mathcal{S}$ (generated by $\mathcal{C}$ and $\mathcal{L}$) is the surface containing of the set of points on all lines which
- intersect $\mathcal{C}$
- are parallel to $\mathcal{L}$.

Rmk. To visual $\mathcal{S}$, move line $\mathcal{L}$ to parallel lines passing thru $\mathcal{C}$.

$\boxed{\text{A cylinder is one type of a surface.}}$

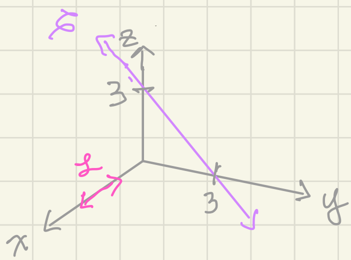
Next : 2 Examples of

Cylinders generated by
- curve $\mathcal{C}$ in an coordinate plane (eg xy-plane)
- line $\mathcal{L}$ is the coord. axis of the unused coord. of $\mathcal{C}$ (eg z-axis)

Ex 1 See Demos 12.1.2.

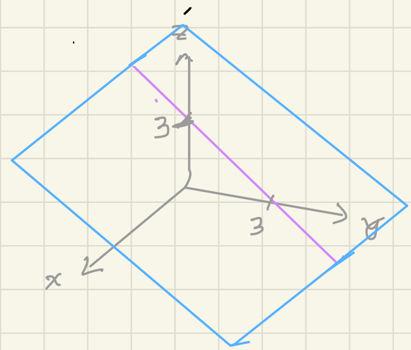
Let $\mathcal{C}$ be $z = 3 - y$ and $\mathcal{L}$ be x-axis.

First graph $\mathcal{C}$. When $x = 0$, $z = 3 - y$ is a line in the yz-plane and goes thru the points $(0, 0, 3)$ and $(0, 3, 0)$.
Second, to generate the cylinder $\mathcal{S}$, move $\mathcal{L}$, w/out changing it's direction, along $\mathcal{C}$



parallel to $\mathcal{L}$

{{ move $\mathcal{L}$ on iPad }}

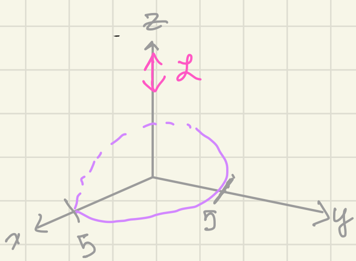


So $\mathcal{S}$ is the plane thru $(0,0,3)$ and $(0,3,0)$ that is parallel to x-axis.

Ex 2. See Demos 12.1.2.

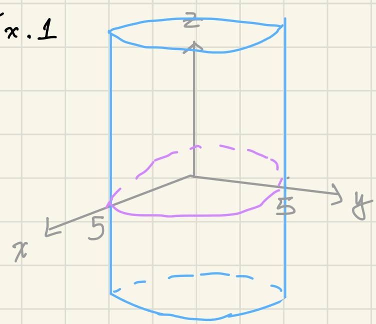
Let $\mathcal{C}$ be $x^2 + y^2 = 25$ and $\mathcal{L}$ be z-axis.

First graph $\mathcal{C}$. Then move $\mathcal{L}$ as we did in Ex. 1



a line $\parallel$ to z-axis, move the line along $\mathcal{C}$ to get $\mathcal{S}$.

{{ move $\mathcal{L}$ on iPad. }}



So $\mathcal{S}$ is a circular cylinder (think of as surface of an infinitely long soup can - just the tin can, not the soup inside.

Quadric Surfaces

- The general equation of a quadric surface (in variables x, y, and z) is where A, B, C, D, E, F, G, H, I, and J are constants,

$$\underbrace{Ax^2 + By^2 + Cz^2}_{\text{"basic terms"}} + \underbrace{Dxy + Exz + Fyz}_{\text{come from rotation}} + \underbrace{Gz + Hy + Iz}_{\text{come from translation}} + J = 0,$$

- We will concentrate on 2 types of quadric surfaces:

hyperbolic paraboloid (commonly called "saddle") and ellipsoid

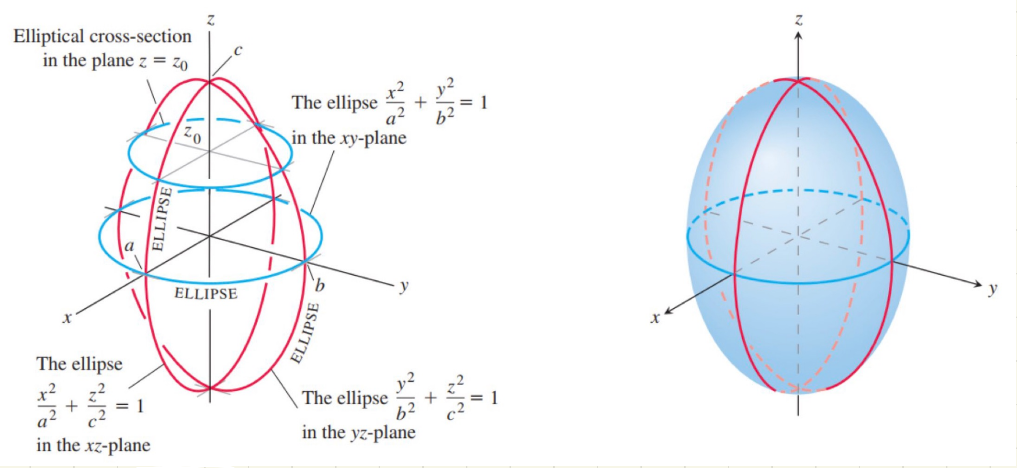
Ellipsoid

- Basic form (a, b, and c are positive)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

multiply thru by $(abc)^2$ to get

$$\overbrace{(bc)^2}^{A} x^2 + \overbrace{(ac)^2}^{B} y^2 + \overbrace{(ab)^2}^{C} z^2 - \overbrace{(abc)^2}^{J} = 0$$

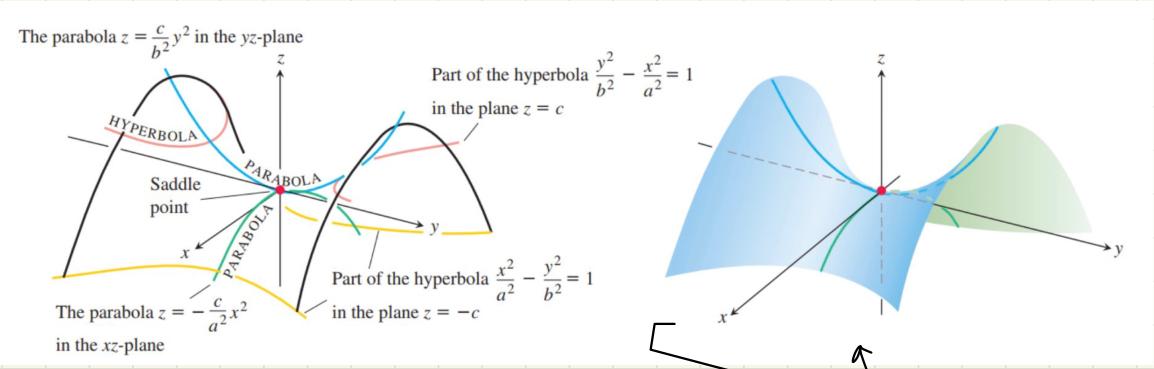


Ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ as center $(0,0,0)$ and semi-axes of length: a (along x-axis), b (along y-axis), c (along z-axis).

Hyperbolic Paraboloid i.e. "Saddle"

A saddle (a, b, and c are positive)

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = \frac{z}{c}$$ ← c, not c^2



colors match up with

- "Saddle point" at the origin.
- Symmetric about the planes $x = 0$ and $y = 0$.
- Cross section with plane $x = 0$ is the parabola $z = \frac{c}{b^2} y^2$.
- Cross section with plane $y = 0$ is the parabola $z = \frac{-c}{a^2} x^2$.
- cross section with plane $z = c$ is the hyperbola $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$

Rmk A saddle with "saddle point" at the origin (and is not rotated) has the equation in variables u, v, and w, for some nonzero constants K and L

$$Ku^2 + Lv^2 = w^1 \text{ with } \left|\frac{K}{K}\right| = -\left|\frac{L}{L}\right|.$$

Problem Describe the surface given by the equation

$$36x^2 + 9y^2 + 4z^2 - 288x - 90y + 48z + 909 = 0 . \qquad (1)$$

- Complete squares in (1) to "get rid of" the x, y, and z terms.

$$(1) \iff 36x^2 - 288x + 9y^2 - 90y + 4z^2 + 48z = -909$$

$$\iff 36(x^2 - 8) + 9(y^2 - 10) + 4(z^2 + 12z) = -909$$

complete square ... if you need, see Ex 4 p 712 - 713

$$\iff 36(x-4)^2 + 9(y-5)^2 + 4(z+6)^2 = -909 + 36(4)^2 + 9(5)^2 + 4(6)^2$$

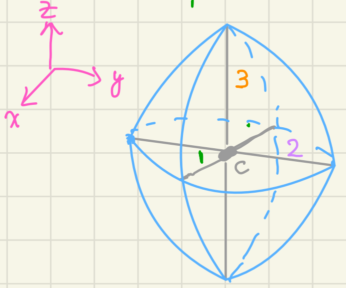
$$\iff 36(x-4)^2 + 9(y-5)^2 + 4(z+6)^2 = 36$$

$36 = (4)(9)$

$$\iff \frac{36(x-4)^2}{36} + \frac{9(y-5)^2}{9 \cdot 4} + \frac{4(z+6)^2}{4 \cdot 9} = 1$$

$$\iff \boxed{\frac{(x-4)^2}{1^2} + \frac{(y-5)^2}{2^2} + \frac{(z-(-6))^2}{3^2} = 1 .} \qquad (2)$$

- An ellipsoid with center $(4, 5, -6) := C$ and



  - a semi-axis $\parallel$ x-axis and of length 1 with corresp. axis btw $(4-1, 5, -6)$ and $(4+1, 5, -6)$
  - a semi-axis $\parallel$ y-axis and of length 2 with corresp. axis btw $(4, 5-2, -6)$ and $(4, 5+2, -6)$
  - a semi-axis $\parallel$ z-axis and of length 3 with corresp. axis btw $(4, 5, -6-3)$ and $(4, 5, -6+3)$