§ 12.6 Cylinders and Quadric Surfaces

- We now consider examples of a surface in $\mathbb{R}^3$ that is given by equations in $x, y,$ and z.

- **Ex 0** We have all ready studied 2 surfaces
  - 0.1 A plane $ax + by + cz = 0$
  - 0.2 A sphere $x^2 + y^2 + z^2 = r^2$.

$\boxed{\text{Cylinders}}$ ← another type of surface.

- With help of Desmos 12.6.1, we look at cylinders in $\mathbb{R}^3$ "generated" by an equation of 2 variables (e.g. x and z) or a function of 1 variable (eg $z = f(x)$ .... so $f(x) - z = 0$).

Four Examples of cylinders: (space provided for you to sketch)

Ex 1. $(x-2)^2 + (z-3)^2 = 1$ (cirular) Cylinder

Ex 2: $y^2 + z^2 = 4$ (Cirular) Cylinder

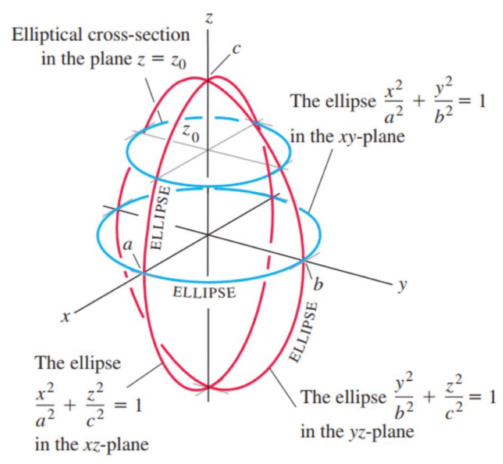
Ex 3: $-x^2 + y = 1$ (note $\Leftrightarrow$ $y = 1 + x^2$) (Parabolic) Cylinder

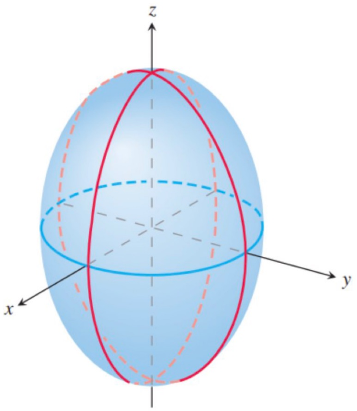
Ex 4: $z = \sin y$ (Sine) Cylinder

$\boxed{\text{Ellipsoid}}$ ← another type of surface.

- Basic form (a, b, and c are positive constants)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$ ← multiply thru by $(abc)^2$ to get $(bc)^2 x^2 + (ac)^2 y^2 + (ab)^2 z^2 = (abc)^2$





Ellipsoid $\quad \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad$ has:

- center $(0,0,0)$

and

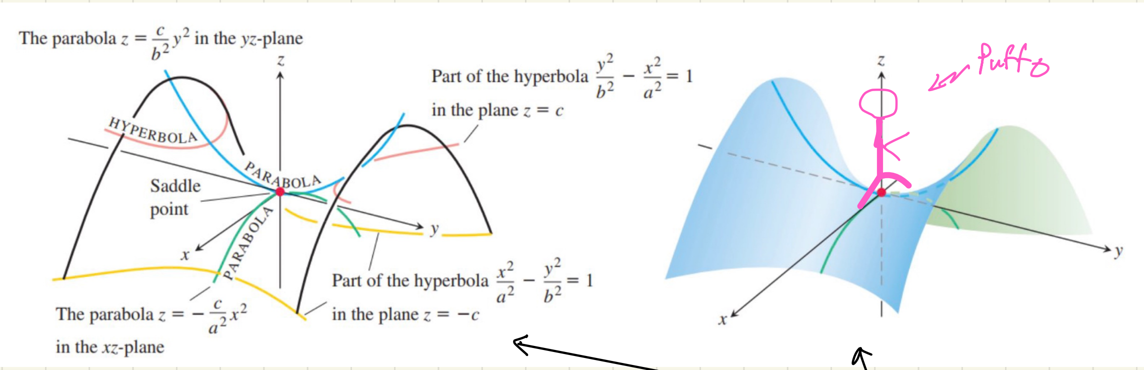
- three semi-axes of length:
  - a (along x-axis),
  - b (along y-axis),
  - c (along z-axis)

Hyperbolic Paraboloid
i.e. "Saddle"

← another type of surface.

<u>A saddle</u> (a, b, and c are positive constants)

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = \frac{z}{c} \leftarrow c, \text{ not } c^2$$



colors match up with

- "Saddle point" at the origin.
- Symmetric about the planes $x=0$ and $y=0$.
- Cross section with plane $x=0$ is the parabola $z = \frac{c}{b^2} y^2$.
- Cross section with plane $y=0$ is the parabola $z = \frac{-c}{a^2} x^2$.
- cross section with plane $z=c$ is the hyperbola $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$
- Note: If a puffo is sitting in the saddle

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = \frac{z}{c}$$

then
  - his head is pointing up the z-axis
  - his legs are hanging in the xz-plane (i.e. $y=0$) over the $\underline{\text{CCD}}$ parabola $z = -\frac{c}{a^2} x^2$.

Problem Describe the surface given by the equation

$$36x^2 + 9y^2 + 4z^2 - 288x - 90y + 48z + 909 = 0 . \qquad (1)$$

- Complete squares in (1) to "get rid of" the $x, y,$ and z terms.

$$(1) \iff 36x^2 - 288x + 9y^2 - 90y + 4z^2 + 48z = -909$$

$$\iff 36(x^2 - 8x) + 9(y^2 - 10y) + 4(z^2 + 12z) = -909$$

complete square ... if you need, see Ex4 p 712 - 713

$$\iff 36(x-4)^2 + 9(y-5)^2 + 4(z+6)^2 = -909 + 36(4)^2 + 9(5)^2 + 4(6)^2$$

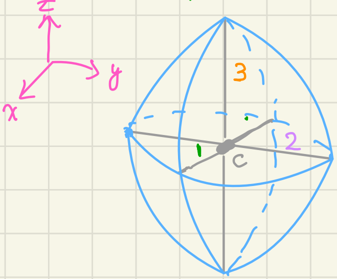
$$\iff 36(x-4)^2 + 9(y-5)^2 + 4(z+6)^2 = 36$$

$36 = (4)(9)$

$$\iff \frac{36(x-4)^2}{36} + \frac{9(y-5)^2}{9 \cdot 4} + \frac{4(z+6)^2}{4 \cdot 9} = 1$$

$$\iff \boxed{\frac{(x-4)^2}{1^2} + \frac{(y-5)^2}{2^2} + \frac{(z-(-6))^2}{3^2} = 1 .} \qquad (2)$$

- An ellipsoid with center $(4, 5, -6) := C$ and



  - a semi-axis $\parallel$ x-axis and of length 1 with corresp. axis btw $(4-1, 5, -6)$ and $(4+1, 5, -6)$
  - a semi-axis $\parallel$ y-axis and of length 2 with corresp. axis btw $(4, 5-2, -6)$ and $(4, 5+2, -6)$
  - a semi-axis $\parallel$ z-axis and of length 3 with corresp. axis btw $(4, 5, -6-3)$ and $(4, 5, -6+3)$