12.5 Lines and Planes (in 3D)

Recall the def normal = $\perp$ = perpendicular

12.5.1

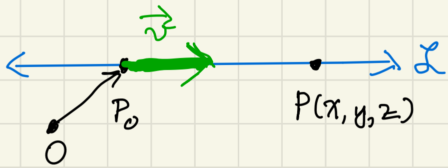
Summary of Equations

TL = Thinking Land.

Line $\mathcal{L}$	Plane $\mathcal{P}$
thru $P_0 = (x_0, y_0, z_0)$	thru $P_0 = (x_0, y_0, z_0)$
in direction of $\vec{v} = \langle a,b,c \rangle \neq 0$	with normal $\vec{n} = \langle a,b,c \rangle \neq \vec{0}$

where $P = (x,y,z)$ is an arbitrary point on

$P \in \mathcal{L}$



TL: Start at origin O, walk to P_0, then walk in +/− direction of $\vec{v}$ for time t.

So: for $-\infty < t < \infty$ get:

How to remember equation

$$\vec{R}(t) = \vec{OP_0} + t\,\vec{v} \qquad (1)$$

Vector Eq.

$$\vec{R}(t) = \langle x_0, y_0, z_0 \rangle + t\langle a,b,c \rangle$$

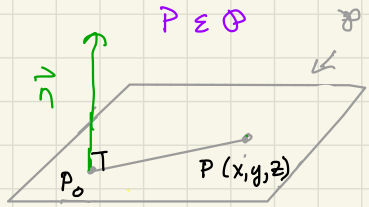
i.e.

$$\vec{R}(t) = \langle x_0 + at,\ y_0 + bt,\ z_0 + ct \rangle$$

Parametric Eq.

$$x(t) = x_0 + a\,t$$
$$y(t) = y_0 + b\,t$$
$$z(t) = z_0 + c\,t,$$

$P \in \mathcal{P}$



How to remember equation

$$\vec{n} \cdot \vec{P_0P} = 0$$

Vector Eq.

$$\langle a,b,c \rangle \cdot \langle x - x_0,\ y - y_0,\ z - z_0 \rangle = 0$$

Component Eq.

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Component Eq. simplified

$$ax + by + cz = d$$

where $d = ax_0 + by_0 + cz_0 \in \mathbb{R}$

Note If $abc \neq 0$, parametric eq. is equivalent to (solve for t) the 2 equations

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}.$$

Think of (1) as:

$$\vec{R}(t) = \underbrace{\vec{OP_0}}_{\text{initial position}} + \underbrace{t}_{\text{time}}\,\underbrace{\|\vec{v}\|}_{\text{speed}}\,\underbrace{\frac{\vec{v}}{\|\vec{v}\|}}_{\substack{\text{direction} \\ \text{(a unit vector)}}}$$

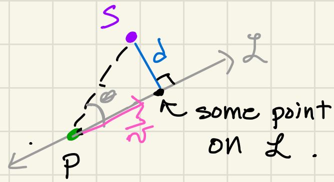
Distance btw a point S and a line/plane

Line $\mathcal{L}$

The distance d from a point S to the line $\mathcal{L}$ thru point P and parallel to $\vec{v}$ is

$$\boxed{d = \left\| \overrightarrow{PS} \times \frac{\vec{v}}{\|\vec{v}\|} \right\|} \overset{\text{note}}{=} \frac{\|\overrightarrow{PS} \times \vec{v}\|}{\|\vec{v}\|}$$

↑ memory form ↗ working form



since

$$\left\| \overrightarrow{PS} \times \frac{\vec{v}}{\|\vec{v}\|} \right\| = \|\overrightarrow{PS}\| \left\| \frac{\vec{v}}{\|\vec{v}\|} \right\| \left| \sin\theta_{\overrightarrow{PS}, \frac{\vec{v}}{\|\vec{v}\|}} \right| \left\| \vec{n}_{\overrightarrow{PS}, \frac{\vec{v}}{\|\vec{v}\|}} \right\|$$

$$= \|\overrightarrow{PS}\| \left| \sin\theta_{\overrightarrow{PS}, \frac{\vec{v}}{\|\vec{v}\|}} \right| = \|\overrightarrow{PS}\| \frac{d}{\|\overrightarrow{PS}\|} = d.$$

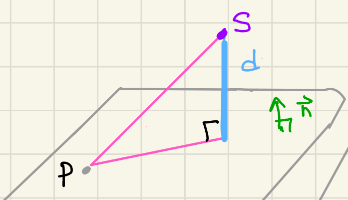
Plane $\mathcal{P}$

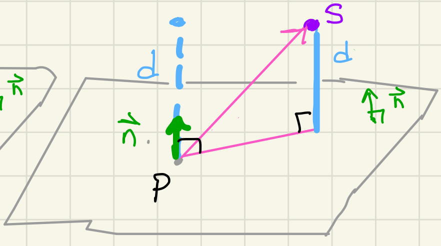
The distance d from a point S to the plane thru the pt. P and with normal $\vec{n}$ is

$$\boxed{d = \left| \overrightarrow{PS} \cdot \frac{\vec{n}}{\|\vec{n}\|} \right|} \overset{\text{note}}{=} \frac{|\overrightarrow{PS} \cdot \vec{n}|}{\|\vec{n}\|}$$

↑ memory form ↗ working form

Picture <not explained in book ☹>





$$d = \|\overrightarrow{\mathrm{proj}}_{\vec{n}} \overrightarrow{PS}\|$$

$$= |\mathrm{comp}_{\vec{n}} \overrightarrow{PS}|$$

$$= \left| \overrightarrow{PS} \cdot \frac{\vec{n}}{\|\vec{n}\|} \right|$$