►. Given vectors
$$\vec{A} = \langle x_A, y_A, z_A \rangle \qquad \text{and} \qquad \vec{B} = \langle x_B, y_B, z_B \rangle .$$

∗. We cannot multiply 2 vectors. We can take the Dot Product, as well as Cross Product, of 2 vectors.

1. For nonzero $\vec{A}$ and $\vec{B}$, let θ_{AB} be the (smallest) angle between $\vec{A}$ and $\vec{B}$. So $\theta_{AB} = \theta_{BA}$.

∘. Dot Product $\vec{A} \cdot \vec{B}$ is a scalar.
$$\vec{A} \cdot \vec{B} \stackrel{\text{def}}{=} x_A x_B + y_A y_B + z_A z_B. \tag{DP$_{\text{def}}$}$$
for nonzero $\vec{A}$ and $\vec{B}$:
$$\vec{A} \cdot \vec{B} = \left\|\vec{A}\right\| \left\|\vec{B}\right\| \cos\theta_{AB} \tag{DP$_\theta$}$$

Def. A normal vector to a plane is any nonzero vector that is perpendicular ($\perp$) to the plane.
Def. Two nonzero vectors are parallel if one is a nonzero scalar multiple of the other. ⟨beware skewed vectors⟩

2. For nonzero, nonparallel $\vec{A}$ and $\vec{B}$, let

$\mathcal{P}_{AB}$ be a plane containing $\vec{A}$ and $\vec{B}$

$\vec{n}_{AB}$ be the right-hand-rule unit normal vector to plane $\mathcal{P}_{AB}$.

Remarks on $\vec{n}_{AB}$.
(2.1) $\vec{n}_{AB}$ is unit $\Leftrightarrow \|\vec{n}_{AB}\| = 1$
(2.2) $\vec{n}_{AB}$ is normal to $\mathcal{P}_{AB} \Leftrightarrow \vec{n}_{AB} \perp \mathcal{P}_{AB} \Leftrightarrow \left[\vec{n}_{AB} \perp \vec{A} \text{ and } \vec{n}_{AB} \perp \vec{B}\right]$.
(2.3) right-hand-rule is to be illustrated in class.
(2.4) How do $\vec{n}_{AB}$ and $\vec{n}_{BA}$ compare? ________________

×. Cross Product $\vec{A} \times \vec{B}$ is a vector.
$$\vec{A} \times \vec{B} \stackrel{\text{def}}{=} \det \begin{bmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ x_A & y_A & z_A \\ x_B & y_B & z_B \end{bmatrix}. \tag{CP$_{\text{def}}$}$$
non-$\|$, nonzero $\vec{A}$ and $\vec{B}$:
$$\vec{A} \times \vec{B} = \left[\left\|\vec{A}\right\| \left\|\vec{B}\right\| \sin\theta_{AB} \right] \vec{n}_{AB}. \tag{CP$_\theta$}$$

3. The determinate in (CP$_{\text{def}}$) gives (just calculate):
(3.1) the cross product of 2 vectors, for which at least one them is $\vec{0}$, is $\vec{0}$
(3.2) the cross product of 2 parallel nonzero vectors is $\vec{0}$.

4. Neat useful facts for nonzero vectors $\vec{A}$ and $\vec{B}$.
(4.1) $\vec{A} \cdot \vec{B} = 0 \quad \Leftrightarrow \quad \vec{A} \perp \vec{B}$ ⟨d**O**t product for **O**thogonal⟩
(4.2) $\vec{A} \times \vec{B} = \vec{0} \quad \Leftrightarrow \quad \vec{A} \parallel \vec{B}$. ⟨unCross the × to form $\|$⟩

5. Cross Product Properties. Given vectors: $\vec{A}$, $\vec{B}$, $\vec{C}$. Given scalars: r, s.
(5.1) $\vec{A} \times \vec{B} = -\left(\vec{B} \times \vec{A}\right)$
(5.2) $\vec{0} \times \vec{B} = \vec{0}$.
(5.3) $\left(r\vec{A}\right) \times \left(s\vec{B}\right) = (rs)\left(\vec{A} \times \vec{B}\right)$
(5.4) $\vec{A} \times \left(\vec{B} + \vec{C}\right) = \left(\vec{A} \times \vec{B}\right) + \left(\vec{A} \times \vec{C}\right)$
(5.5) $\left(\vec{A} + \vec{B}\right) \times \vec{C} = \left(\vec{A} \times \vec{C}\right) + \left(\vec{B} \times \vec{C}\right)$
(5.6) $\vec{A} \times \left(\vec{B} \times \vec{C}\right) \stackrel{(*)}{=} \left(\vec{A} \cdot \vec{C}\right)\vec{B} - \left(\vec{A} \cdot \vec{B}\right)\vec{C}$ (cross product is not associative)
(5.7) $\left(\vec{A} \times \vec{B}\right) \cdot \vec{C} \stackrel{(*)}{=} \vec{A} \cdot \left(\vec{B} \times \vec{C}\right)$ (triple scalar product)

A $\stackrel{(*)}{=}$ indicated that you do not need to memorize for exam but should be able to use if given.