

# Projections and Components

12.3.3

• Fact 9 (a calculation)

$$\frac{\vec{A} \cdot \vec{B}}{\|\vec{B}\|} = \frac{1}{\|\vec{B}\|} (\vec{A} \cdot \vec{B}) = \frac{1}{\|\vec{B}\|} (\|\vec{A}\| \|\vec{B}\| \cos \theta_{AB}) = \|\vec{A}\| \cos \theta_{AB}$$

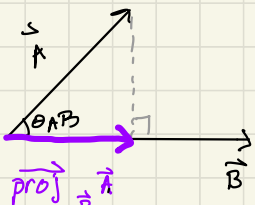
• Defs Let  $\vec{A}$  and  $\vec{B}$  be non zero. (see book p 729)

1. The vector projection of  $\vec{A}$  onto  $\vec{B}$  is denoted  $\text{proj}_{\vec{B}} \vec{A}$  or  $\text{proj}_{\vec{B}} \vec{A}$ .

2. The (signed) scalar component of  $\vec{A}$  in the direction of  $\vec{B}$  is denoted  $\text{comp}_{\vec{B}} \vec{A}$ .

▷ Defs by pictures

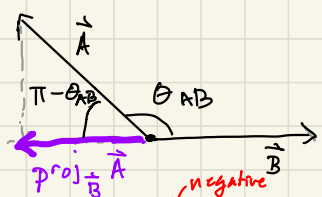
When  $0 \leq \theta_{AB} < \pi/2$



$$\text{comp}_{\vec{B}} \vec{A} = \|\text{proj}_{\vec{B}} \vec{A}\| > 0$$

$$\cos \theta_{AB} = \frac{\text{adj}}{\text{hyp}} = \frac{\|\text{proj}_{\vec{B}} \vec{A}\|}{\|\vec{A}\|}$$

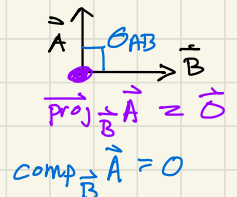
When  $\pi/2 < \theta_{AB} \leq \pi$



$$\text{comp}_{\vec{B}} \vec{A} = -\|\text{proj}_{\vec{B}} \vec{A}\| < 0$$

$$\cos \theta_{AB} = -\cos(\pi - \theta_{AB}) = -\frac{\|\text{proj}_{\vec{B}} \vec{A}\|}{\|\vec{A}\|}$$

When  $\theta_{AB} = \pi/2$



$$\text{proj}_{\vec{B}} \vec{A} = \vec{0}$$

$$\text{comp}_{\vec{B}} \vec{A} = 0$$

$$\cos \theta_{AB} = 0$$

Key observations

- (i)  $\text{comp}_{\vec{B}} \vec{A}$  is  $\begin{cases} > 0, & \text{if } 0 < \theta_{AB} < \pi/2 \\ 0, & \text{if } \theta_{AB} = \pi/2 \\ < 0, & \text{if } \pi/2 < \theta_{AB} \leq \pi \end{cases}$
- (ii) the direction of  $\text{proj}_{\vec{B}} \vec{A}$  is either + or - the direction of  $\frac{\vec{B}}{\|\vec{B}\|}$ .

length of vector  $\rightarrow$  abs. value  $\rightarrow$

$$(ii) \|\text{proj}_{\vec{B}} \vec{A}\| = |\text{comp}_{\vec{B}} \vec{A}|$$

⊛ so get  $\text{proj}_{\vec{B}} \vec{A} = \left( \frac{\text{comp}_{\vec{B}} \vec{A}}{1} \right) \frac{\vec{B}}{\|\vec{B}\|}$

⤴ (1st do case  $0 < \theta_{AB} < \pi/2$  do not dwell on)

a unit vector - direction is either + or - dir. of  $\text{proj}_{\vec{B}} \vec{A}$

▷ Defs to calculate with.

$$\text{proj}_{\vec{B}} \vec{A} \stackrel{\text{def}}{=} \left( \vec{A} \cdot \frac{\vec{B}}{\|\vec{B}\|} \right) \frac{\vec{B}}{\|\vec{B}\|} \stackrel{\text{use Fact 9}}{=} \left( \|\vec{A}\| \cos \theta_{AB} \right) \frac{\vec{B}}{\|\vec{B}\|}$$

$$\text{comp}_{\vec{B}} \vec{A} \stackrel{\text{def}}{=} \left( \vec{A} \cdot \frac{\vec{B}}{\|\vec{B}\|} \right) \stackrel{\text{use Fact 9}}{=} \|\vec{A}\| \cos \theta_{AB}$$