⚠. Beware. On MML the scalar component of $\vec{A}$ in the direction of $\vec{B}$ is denoted 2 different ways:

$$\boxed{\operatorname{comp}_{\vec{B}} \vec{A} \quad = \quad \operatorname{scal}_{\vec{B}} \vec{A}}\,. \tag{⚠}$$

Material from this handout comes from: class lectures and §12.3+12.4 Overview Handout.

►. Given vectors

$$\vec{A} = \langle\, x_A, y_A, z_A \,\rangle \qquad \text{and} \qquad \vec{B} = \langle\, x_B, y_B, z_B \,\rangle\,.$$

1. The $\vec{A}$ dot product $\vec{B}$ is the scalar

$$\vec{A}\cdot\vec{B} \overset{\text{def}}{=} x_A x_B + y_A y_B + z_A z_B. \tag{DP$_{\text{def}}$}$$

For nonzero vectors $\vec{A}$ and $\vec{B}$ ⟨so θ_{AB} makes sense⟩

$$\vec{A}\cdot\vec{B} \;=\; \left\|\vec{A}\right\|\,\left\|\vec{B}\right\|\,\cos\theta_{AB} \tag{DP$_\theta$}$$

and so

$$\cos\theta_{AB} \;\underset{(\text{DP}_\theta)}{\overset{\text{by}}{=}}\; \frac{\vec{A}\cdot\vec{B}}{\left\|\vec{A}\right\|\,\left\|\vec{B}\right\|} \;\underset{(2.2)}{\overset{\text{by}}{=}}\; \frac{\vec{A}}{\left\|\vec{A}\right\|}\cdot\frac{\vec{B}}{\left\|\vec{B}\right\|} \tag{cos}$$

2. Dot Product Properties ⟨for scalars r and s⟩

$$\vec{A}\cdot\vec{A} \;=\; \left\|\vec{A}\right\|^2 \tag{2.1}$$

$$\left(r\vec{A}\right)\cdot\left(s\vec{B}\right) \;=\; (rs)\,\left(\vec{A}\cdot\vec{B}\right) \tag{2.2}$$

3. Vector Projection and Scalar Component onto a nonzero $\vec{B}$

The vector projection of $\vec{A}$ onto $\vec{B}$ is

$$\overrightarrow{\operatorname{proj}}_{\vec{B}}\,\vec{A} \;\overset{\text{def}}{=}\; \left(\vec{A}\cdot\frac{\vec{B}}{\left\|\vec{B}\right\|}\right)\frac{\vec{B}}{\left\|\vec{B}\right\|} \;\underset{(2.2)}{\overset{\text{by}}{=}}\; \left(\frac{\vec{A}\cdot\vec{B}}{\left\|\vec{B}\right\|^2}\right)\vec{B} \;\underset{(2.1)}{\overset{\text{by}}{=}}\; \left(\frac{\vec{A}\cdot\vec{B}}{\vec{B}\cdot\vec{B}}\right)\vec{B}\,. \tag{3.3}$$

The scalar component of $\vec{A}$ in the direction of $\vec{B}$ is

$$\operatorname{comp}_{\vec{B}}\vec{A} \;\overset{\text{def}}{=}\; \vec{A}\cdot\frac{\vec{B}}{\left\|\vec{B}\right\|} \;\underset{(2.2)}{\overset{\text{by}}{=}}\; \frac{\vec{A}\cdot\vec{B}}{\left\|\vec{B}\right\|} \;\underset{\text{denoted}}{\overset{\text{also}}{=}}\; \operatorname{scal}_{\vec{B}}\vec{A}\,. \tag{3.4}$$

Note $\left\|\overrightarrow{\operatorname{proj}}_{\vec{B}}\,\vec{A}\right\| \;=\; \left|\operatorname{comp}_{\vec{B}}\vec{A}\right|$. For nonzero $\vec{A}$ and $\vec{B}$, multiplying (cos) thru by $\left\|\vec{A}\right\|$ gives

$$\overrightarrow{\operatorname{proj}}_{\vec{B}}\,\vec{A} \;\underset{\text{def}}{\overset{\text{by}}{=}}\; \left(\vec{A}\cdot\frac{\vec{B}}{\left\|\vec{B}\right\|}\right)\frac{\vec{B}}{\left\|\vec{B}\right\|} \;\underset{(\cos)}{\overset{\text{by}}{=}}\; \left(\left\|\vec{A}\right\|\,\cos\theta_{AB}\right)\frac{\vec{B}}{\left\|\vec{B}\right\|} \tag{3.5}$$

$$\operatorname{comp}_{\vec{B}}\vec{A} \;\underset{\text{def}}{\overset{\text{by}}{=}}\; \vec{A}\cdot\frac{\vec{B}}{\left\|\vec{B}\right\|} \;\underset{(\cos)}{\overset{\text{by}}{=}}\; \left\|\vec{A}\right\|\,\cos\theta_{AB}\,. \tag{3.6}$$

If $0 \le \theta_{AB} \le \frac{\pi}{2}$ then $\cos\theta_{AB} \ge 0$ so $\operatorname{comp}_{\vec{B}}\vec{A} \ge 0$. If $\frac{\pi}{2} \le \theta_{AB} \le \pi$ then $\cos\theta_{AB} \le 0$ so $\operatorname{comp}_{\vec{B}}\vec{A} \le 0$.